Drawing Interaction Diagram.

نسألكم الدعاء

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اذا حملت تطبيق **RC Structures** YASSER EL LEATHY على تليفونك المحمول او اللوح السطحى ستستطيع أن تشغل أفلام شرح للمقاطع التى تحتوى على رمز

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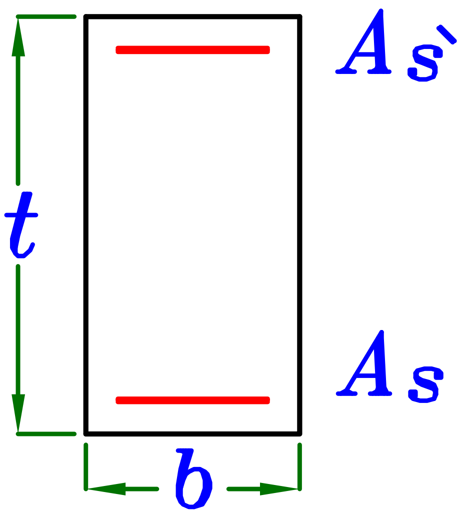
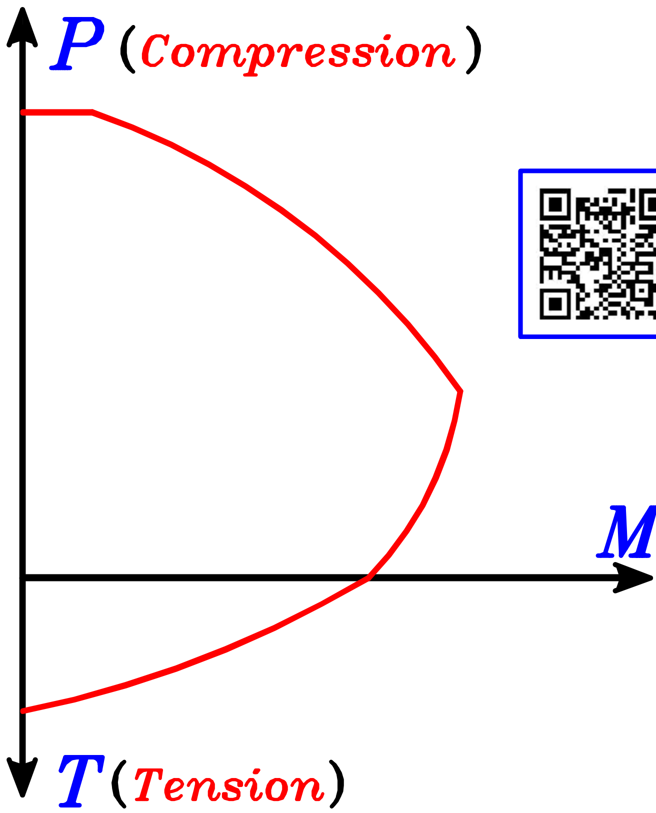
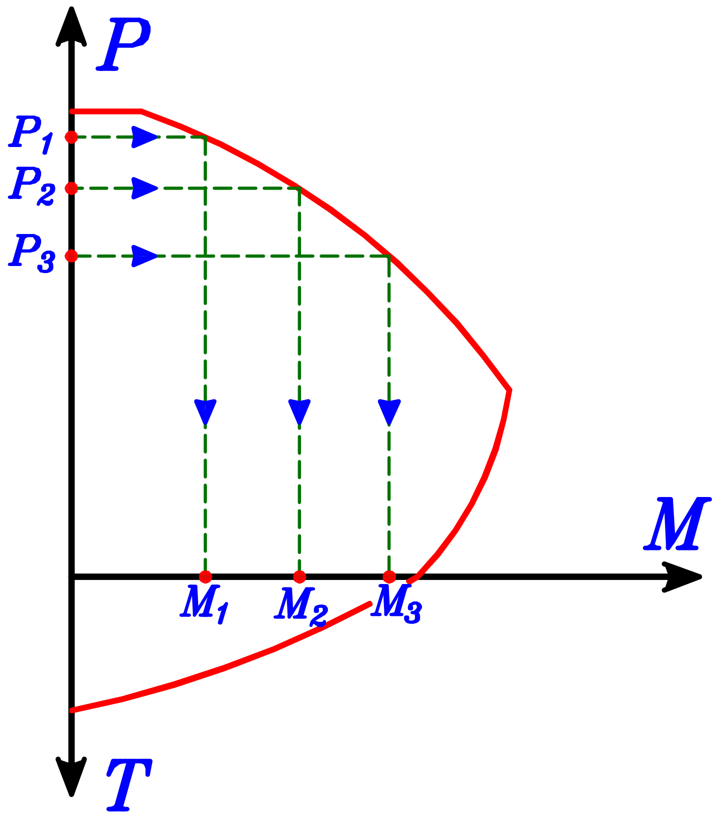
Introduction.

ال *Interaction Diagram* هو رسم بيانى يحدد العلاقه بين أكبر *Normal* & *Moment* يتحملهم قطاع معاً.





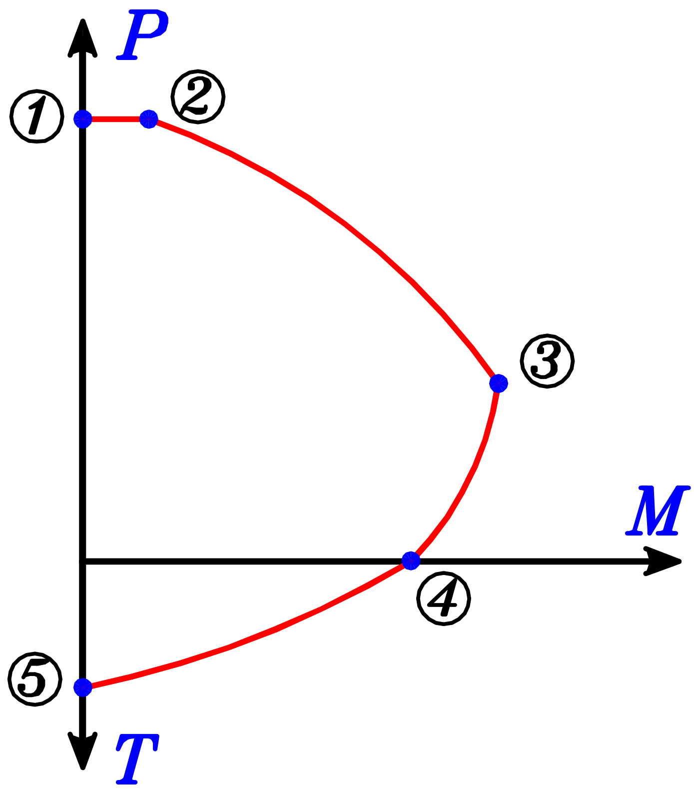
كلما تغير قيمه ال *Normal* المؤثره على القطاع كلما تغيرت قيمه أكبر *moment* يتحملها هذا القطاع .



Important Points in Interaction Diagram.

توجد ٥ نقاط مهمه تحدد شكل ال Interaction Diagram

و هى النقاط التى يتغير عندها ميل المنحنى .



Point 1 Pure Compression (Axially Loaded Column)

$$P = P_{U.L.} \quad , \quad M = Zero \quad , \quad e = \frac{M}{P} = Zero$$

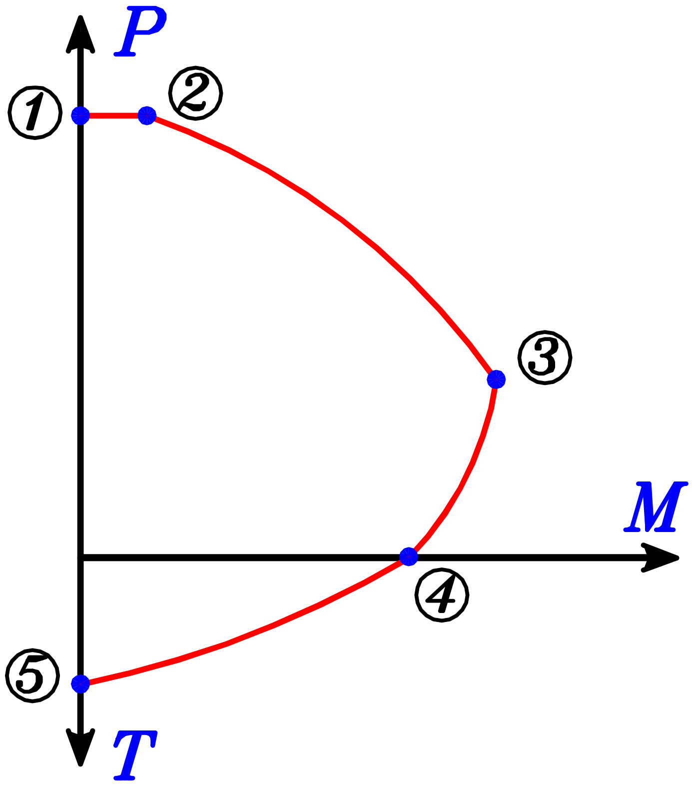
عند هذه النقطه لا يوجد moment مؤثر على القطاع

و نحسب اكبر normal يتحمله هذا القطاع

Point 2 min. eccenterecity. $\boxed{e_{min} = 0.05\ t}$

$$P = P_{U.L.} \quad , \quad M = P_{U.L.} * e_{min} \quad , \quad e_{min} = \frac{M}{P} = 0.05\ t$$

عند هذه النقطه يوجد moment قليل جدا من الممكن اهمال تأثيره عند التصميم .



Point 3 Balanced Section. $\boxed{e=e_b}$

$$P=P_b \quad , \quad M=M_b \quad , \quad e=e_b=\frac{M_b}{P_b}$$

عند هذه النقطه عند حدوث انهيار يكون القطاع Balanced Section

و هو القطاع الفاصل بين Compression Failure & Tension Failure

Point 4 Pure bending (as a Beams)

$$P=Zero \quad , \quad M=M_{U.L.} \quad , \quad e=\frac{M}{P}=\infty$$

عند هذه النقطه لا يوجد normal مؤثر على القطاع

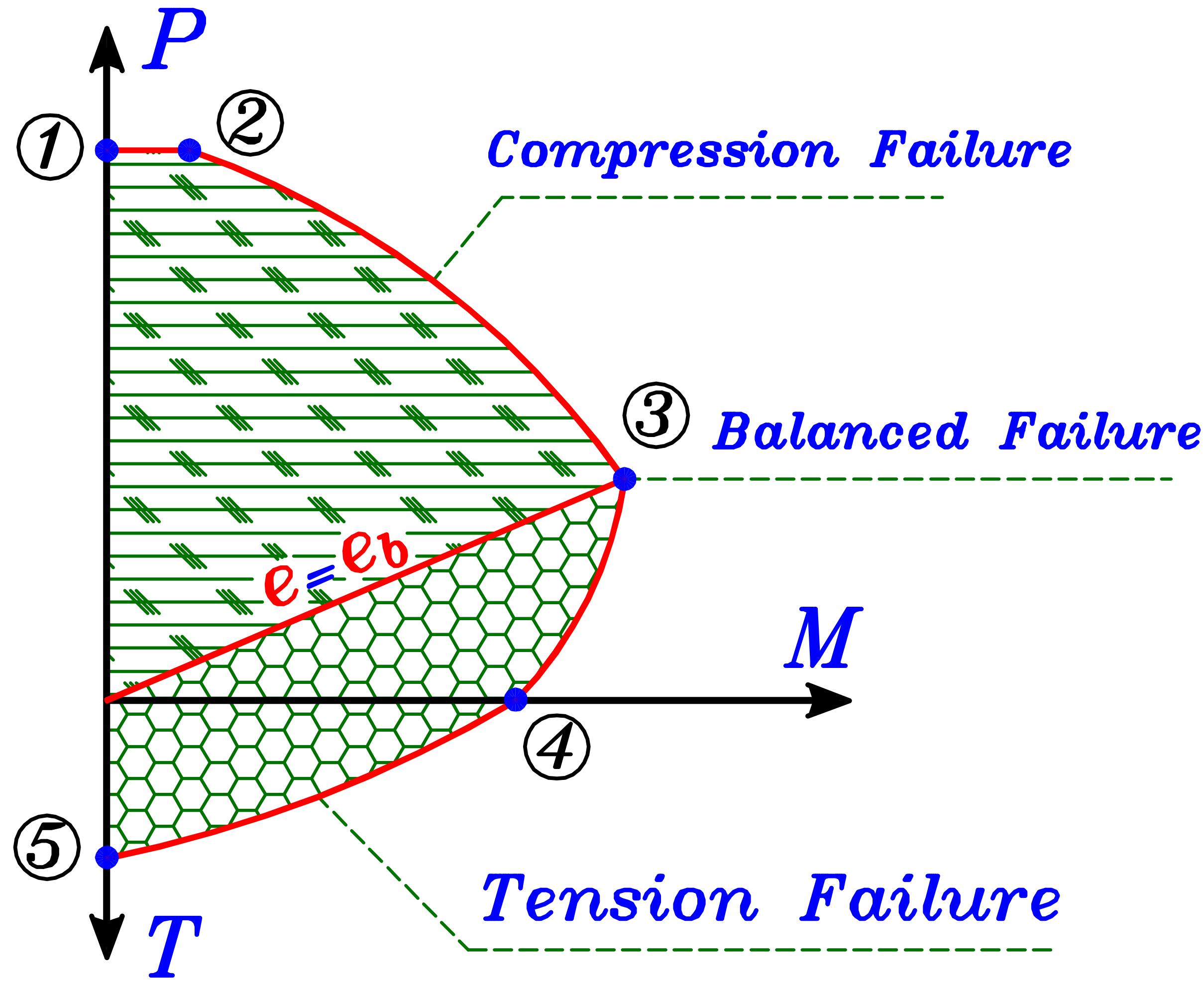
و نحسب اكبر moment يتحمله هذا القطاع

Point 5 Pure Tension (Tie)

$$P=T_{U.L.} \quad , \quad M=Zero \quad , \quad e=\frac{M}{T}$$

عند هذه النقطه لا يوجد moment مؤثر على القطاع

و نحسب اكبر Tension يتحمله هذا القطاع



Eccentricity. (e)

$$e_b = \frac{M_b}{P_b} = \text{eccentricity at balanced Failure.}$$

IF $\frac{e}{t} \leqslant 0.05$ neglect M $\longrightarrow$ Axially Loaded Column.

IF $0.05 < \frac{e}{t} < \frac{e_b}{t}$ Medium eccentricity (Compression Failure)

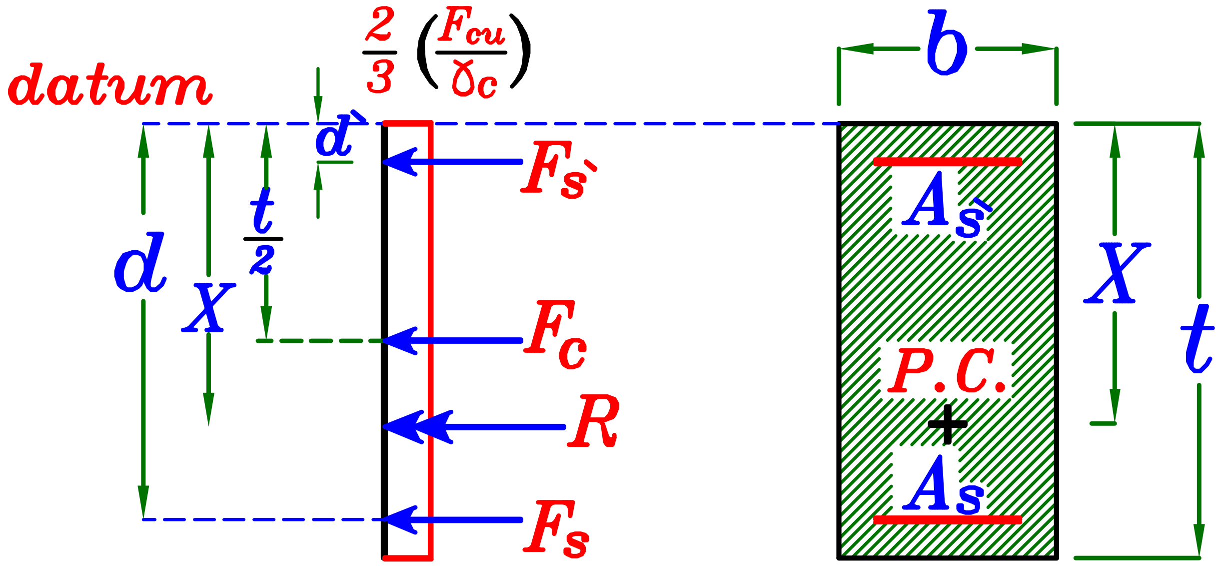
IF $\frac{e}{t} > \frac{e_b}{t}$ Large eccentricity (Tension Failure)

IF $\frac{e}{t} = \infty$ M only $\longrightarrow$ Beams.

Plastic Centroid (P.C.)

فى حاله وجود M,N على القطاع نأخذ العزوم دائما حول نقطة واحده فقط

و هذه النقطة تسمى Plastic Centroid



و لكى نحدد مكان ال P.C. نفرض أن القطاع بأكملة معرض للضغط

و يكون مكان ال P.C. هو مكان محصلة القوى كلها (R).

X = The distance From the Plastic Centroid to the datum.

$$X = \frac{F_s`\,(d`) + F_c\left(\frac{t}{2}\right) + F_s(d)}{F_s` + F_c + F_s}$$

$$X = \frac{A_s`\left(\frac{F_y}{\delta_s}\right)(d`) + \frac{2}{3}\left(\frac{F_{cu}}{\delta_c}\right) b\,t\left(\frac{t}{2}\right) + A_s\left(\frac{F_y}{\delta_s}\right)(d)}{A_s`\left(\frac{F_y}{\delta_s}\right) + \frac{2}{3}\left(\frac{F_{cu}}{\delta_c}\right) b\,t + A_s\left(\frac{F_y}{\delta_s}\right)}$$

ممكن للتسهيل أخذ مكان ال P.C. فى منتصف القطاع أى أن $\boxed{X = \frac{t}{2}}$

* F.O.S. For Materials.

1- Case of bending moment only (M) or Tension only (T)
or Axial tension & bending moment $(M+T)$
or Shear (Q) only or Torsion only (M_t) or Shear & Torsion $(Q+M_t)$

$$\boxed{\gamma_c = 1.5 \quad , \quad \gamma_s = 1.15}$$ ✓✓

2- Case of Axial compression Force only. (P).

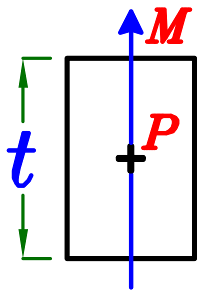
$$\boxed{\gamma_c = 1.75 \, , \quad \gamma_s = 1.34}$$

3- Case of Axial compression Force and bending moment $(M+P)$

$$\boxed{e = \frac{M}{P}}$$

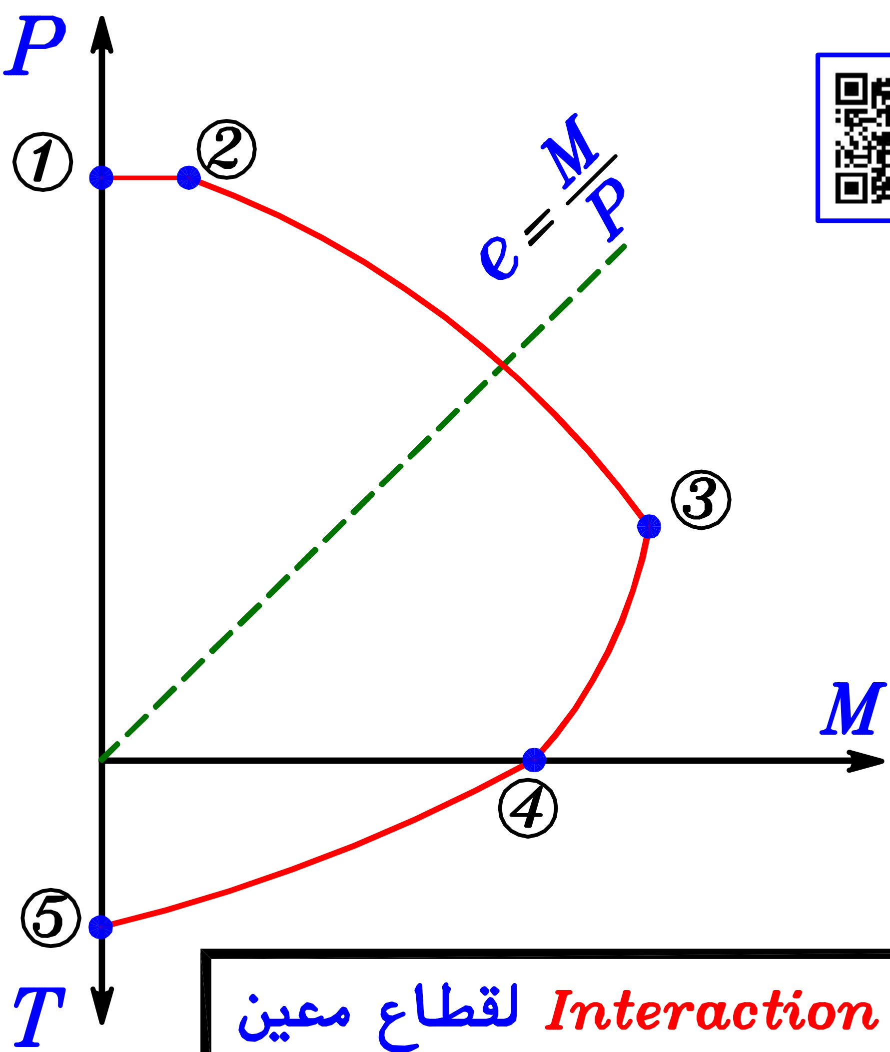
$$\gamma_c \text{ (Concrete)} = 1.5 \left[\left(\frac{7}{6}\right) - \frac{(e\backslash t)}{3}\right] \geq 1.5$$

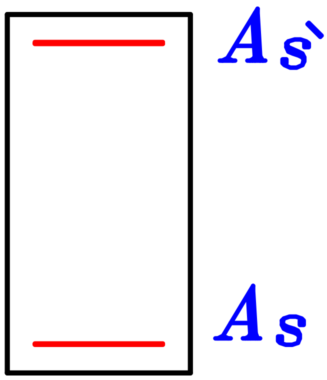
$$\gamma_s \text{ (Steel)} = 1.15 \left[\left(\frac{7}{6}\right) - \frac{(e\backslash t)}{3}\right] \geq 1.15$$



$\therefore$ Allowable stress For concrete. $= \dfrac{F_{cu}}{\gamma_c}$

Allowable stress For steel. $= \dfrac{F_y}{\gamma_s}$





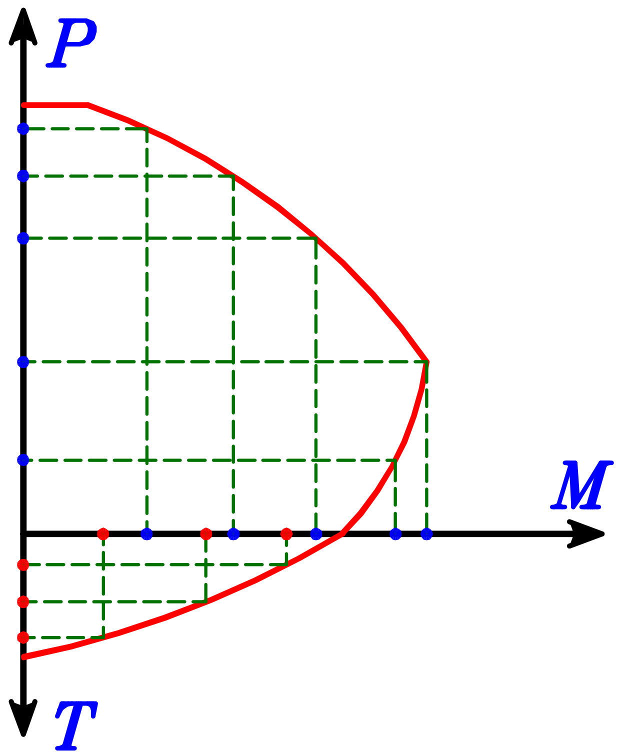
شكل ال *Interaction Diagram* لقطاع معين

ملحوظة هامه

نستطيع تحديد اذا كان القطاع *Safe or Unsafe* تحت تأثير القوى المختلفة و ذلك برسم ال *Interaction Diagram* و ندخل بقيمة ال *M,P* المؤثرتان على هذا القطاع

- اذا كانت نقطة تقاطع كلا من *M,P* واقعه على المنحنى أو بداخله يكون القطاع *Safe*
- اذا كانت نقطة تقاطع كلا من *M,P* واقعه خارج المنحنى يكون القطاع *UnSafe*

لرسم الـ Interaction Diagram نحتاج لحساب عدد لا نهائى من النقاط



و بالطبع لن نستطيع أن نحسب عدد لا نهائى من النقاط
لذا سنحسب عدد من النقاط ليعطينا شكل قريب من الـ Interaction Diagram

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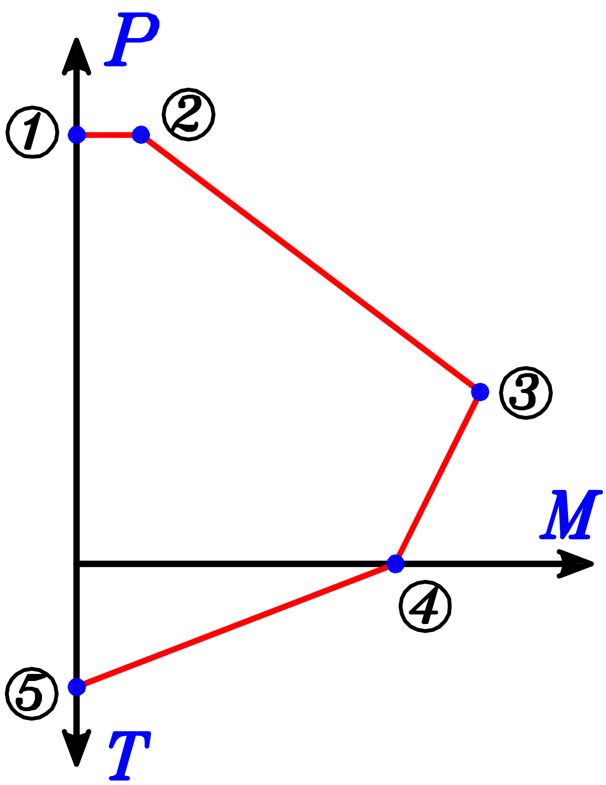
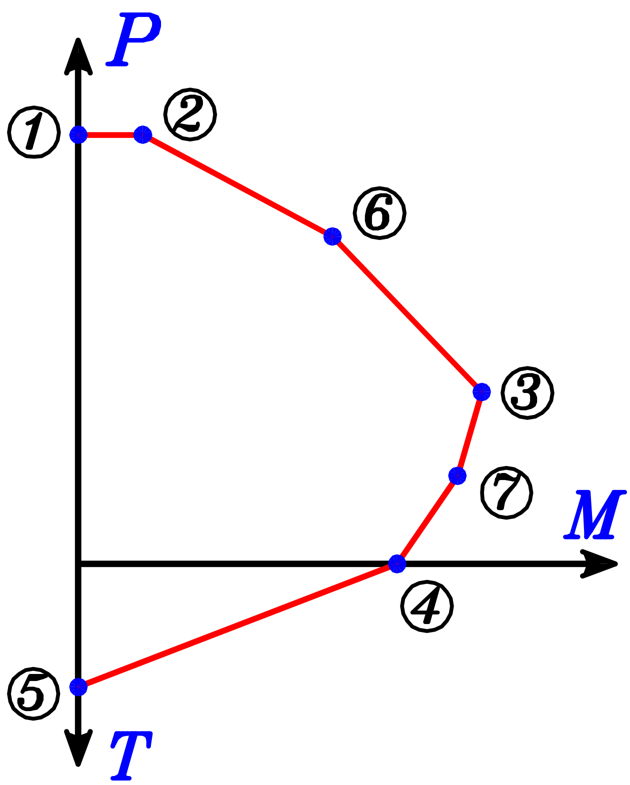
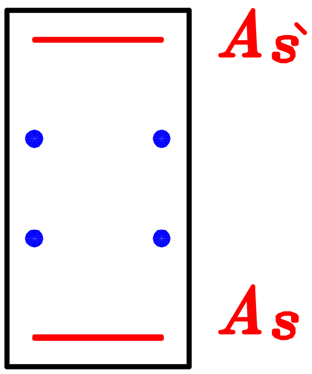
Simplified Interaction Diagram
5 Point only

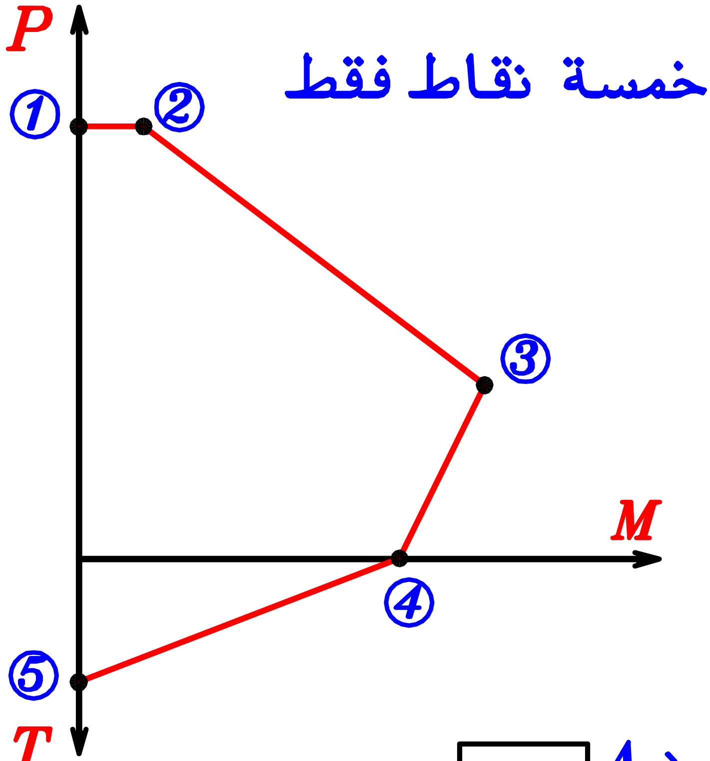


Exact Interaction Diagram
7 Point only



Drawing Simplified Interaction Diagram.
By using Five Points only.

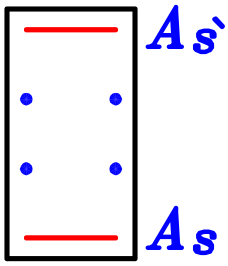




Point 1

$$P = P_{U.L.} = 0.35\, A_c\, F_{cu} + 0.67\, A_s\, F_y$$

$$M = Zero$$

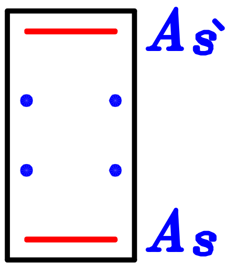


A_s هي مساحه الحديد الكلي للقطاع

$A_s = A_s + A_s$` + مساحه الحديد الجانبي

Point 2

min. eccentricity $e_{min.} = 0.05\, t$

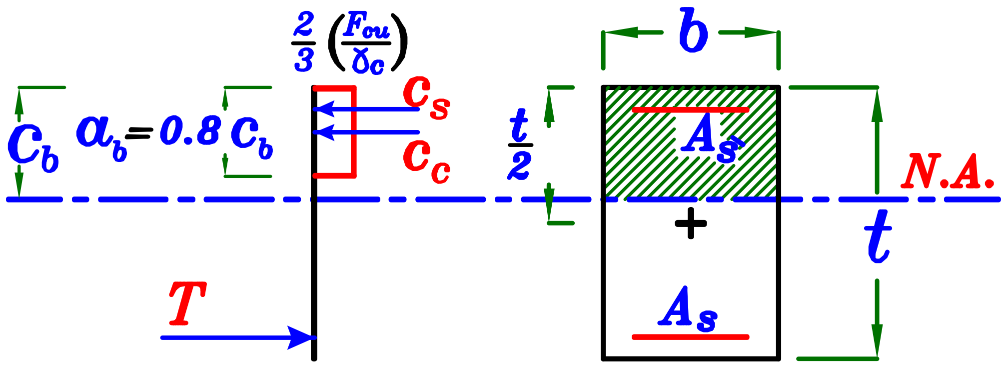


$$P = P_{U.L.} = 0.35\, A_c\, F_{cu} + 0.67\, A_s\, F_y$$

$$M = P_{U.L.} * e_{min.} = P_{U.L.} * 0.05\, t$$

A_s هي مساحه الحديد الكلي للقطاع

Point ③ (Balanced Failure)



$$P = P_b \;,\quad M = M_b \longrightarrow e = e_b$$

For Balanced Sec. $C = C_b = \left(\dfrac{600}{600 + (F_y \backslash \gamma_s)}\right) * d$

$a = a_b = 0.8\ C_b \quad , \quad F_s = \left(\dfrac{F_y}{\gamma_s}\right)$

$$\boxed{P = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a_b\, b + A_s` \frac{F_y}{\gamma_s} - A_s \frac{F_y}{\gamma_s}} \simeq \boxed{\frac{2}{3}\frac{F_{cu}}{\gamma_c} a_b\, b}$$

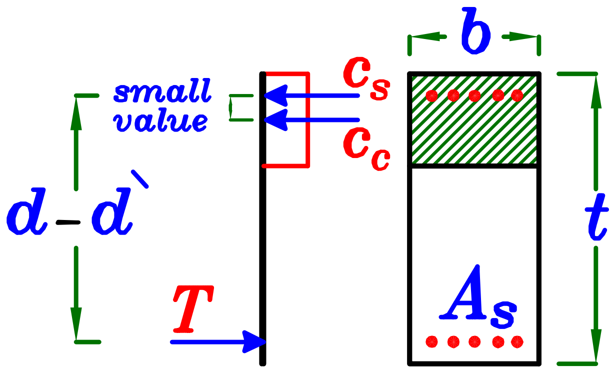
$$\boxed{M = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a_b\, b \left(\frac{t}{2} - \frac{a_b}{2}\right) + A_s` \frac{F_y}{\gamma_s}\left(\frac{t}{2} - d`\right) + A_s \frac{F_y}{\gamma_s}\left(\frac{t}{2} - c\right)}$$

A_s هى مساحه حديد الشد فقط و لا نأخذ الحديد الجانبى فى الحسابات

Point ④

$$\boxed{P = Zero}$$

$$\boxed{M = A_s \frac{F_y}{\gamma_s} (d - d`)}$$



A_s هى مساحه حديد الشد فقط و لا نأخذ الحديد الجانبى فى الحسابات

Point ⑤

$$\boxed{T = A_s * \frac{F_y}{\gamma_s}} \qquad \boxed{M = Zero}$$

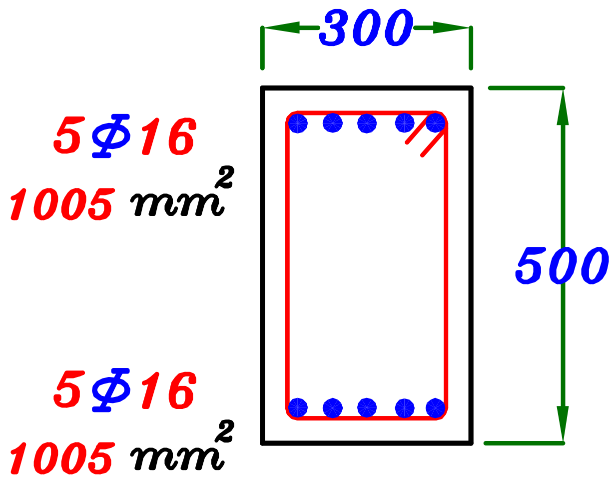
A_s هى مساحه الحديد الكلى للقطاع

$$\boxed{A_s = A_s + A_s` + \text{مساحه الحديد الجانبى}}$$

Example.

$F_{cu} = 30\ N\backslash mm^2$

$F_y = 400\ N\backslash mm^2$



Req.

① Draw the Interaction Diagram

(Using Five points only)

② Find: ⓐ M_U at $P_U = 1400\ kN$

ⓑ P_U at $M_U = 220\ kN.m$

ⓒ M_U , P_U at Eccentricity $= 150\ mm$

③ Find IF the Sec. is safe or not.

when: ⓔ $M_U = 125.4\ kN.m$, $P_U = 1692\ kN$

ⓕ $M_U = 112.2\ kN.m$, $P_U = 807.2\ kN$

ⓖ $M_U = 244.8\ kN.m$, $P_U = 1365\ kN$

ⓗ $M_U = 43.0\ kN.m$, $T_U = 290.4\ kN$

Solution.

Get P.C. $X = \frac{t}{2} = 250\ mm$

$$C_b = \left(\frac{600}{600 + (F_y \backslash \delta_s)}\right) * d = \left(\frac{600}{600 + \left(\frac{400}{1.15}\right)}\right) * 450 = 284.8\ mm$$

Point ① $M = Zero$

$A_c = 300 * 500 = 150000\ mm^2$

$A_s = (A_s + A_s') = 1005 + 1005 = 2010\ mm^2$

$$P = P_{U.L.} = 0.35\, A_c\, F_{cu} + 0.67\, A_s\, F_y$$

$$= 0.35\ (150000)(30) + 0.67\ (2010)\ (400) = 2113680\ N$$

$$= 2113.680\ kN$$

$$\boxed{P_{U.L.} = 2113.680\ kN\ ,\quad M_{U.L.} = Zero}$$

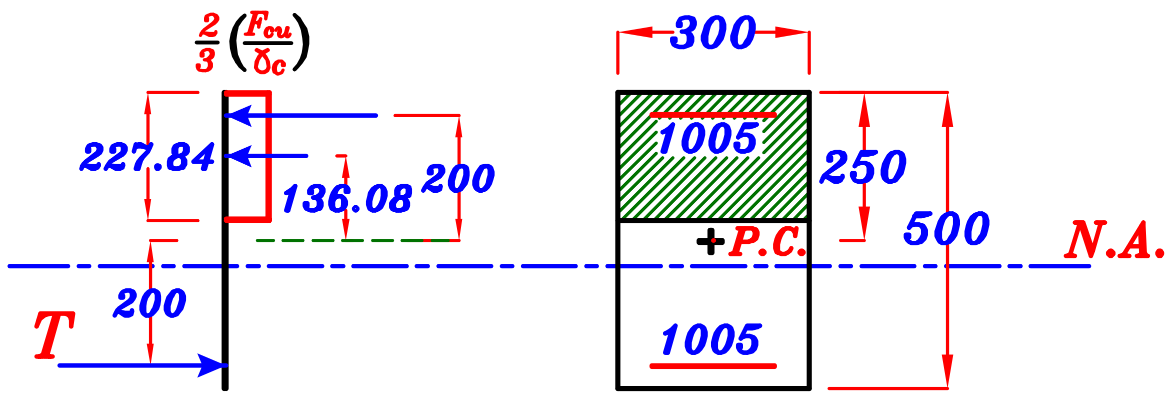
Point ② min. eccentricity $e_{min.} = 0.05\,t$

$$P = P_{U.L.} = 0.35\,A_c\,F_{cu} + 0.67\,A_s\,F_y = 2113680\ N$$
$$= 2113.680\ kN$$

$$M = P_{U.L.} * e_{min.} = P_{U.L.} * 0.05\,t = 2113680 * 0.05(500)$$
$$= 52842000\ N.mm = 52.84\ kN.m$$

$$\boxed{P_{U.L.} = 2113.680\ kN\ ,\ M_{U.L.} = 52.84\ kN.m}$$

Point ③ (Balanced Failure)



Take $C = C_b = 284.8\ mm$ $\quad a_b = 0.8\,C_b = 227.84\ mm$

$$\boxed{P = P_{U.L.} = C_c + C_s - T = \frac{2}{3}\frac{F_{cu}}{\gamma_c}a_b b + A_s^{`}\frac{F_y}{\gamma_s} - A_s\frac{F_y}{\gamma_s}}$$

$$P = \frac{2}{3}\left(\frac{30}{1.5}\right)(227.84)(300) + 1005\left(\frac{400}{1.15}\right) - 1005\left(\frac{400}{1.15}\right) = 911360\ N$$
$$= 911.36\ kN$$

$$\boxed{M = \frac{2}{3}\frac{F_{cu}}{\gamma_c}a_b b\left(\frac{t}{2} - \frac{a_b}{2}\right) + A_s^{`}\frac{F_y}{\gamma_s}\left(\frac{t}{2} - d^{`}\right) + A_s\frac{F_y}{\gamma_s}\left(\frac{t}{2} - c\right)}$$

$$M = \frac{2}{3}\left(\frac{30}{1.5}\right)(227.84)(300)(136.08) + 1005\left(\frac{400}{1.15}\right)(200) + 1005\left(\frac{400}{1.15}\right)(200)$$
$$= 263843955.8\ N.mm = 263.84\ kN.m$$

$$\boxed{P_b = 911.36\ kN\ ,\ M_b = 263.84\ kN.m}$$

Point ④ $N = Zero$

$$M = A_s \frac{F_y}{\gamma_s}(d - d`) = 1005 * \frac{400}{1.15}(450 - 50) = 139826087 \text{ N.mm}$$
$$= 139.82 \text{ kN.m}$$

$$\boxed{P_{U.L.} = Zero \quad , \quad M_{U.L.} = 139.82 \text{ kN.m}}$$

Point ⑤ $M = Zero$

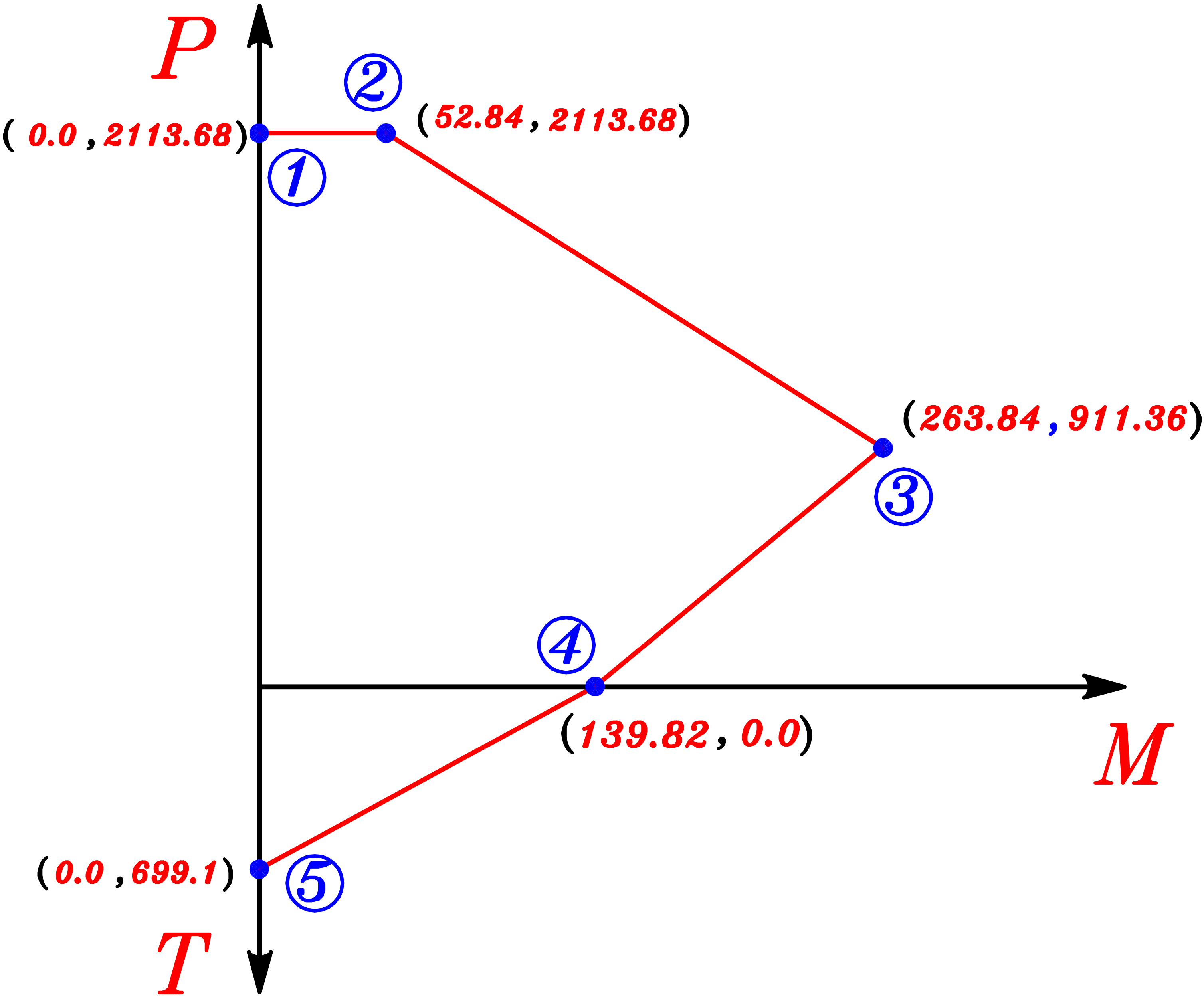
$$A_s = (A_s + A_s`) = 1005 + 1005 = 2010 \text{ mm}^2$$

$$T = A_s * \frac{F_y}{\gamma_s} = 2010 * \frac{400}{1.15} = 699130.4 \text{ N} = 699.1 \text{ kN}$$

$$\boxed{T_{U.L.} = 699.1 \text{ kN} \quad , \quad M_{U.L.} = Zero}$$

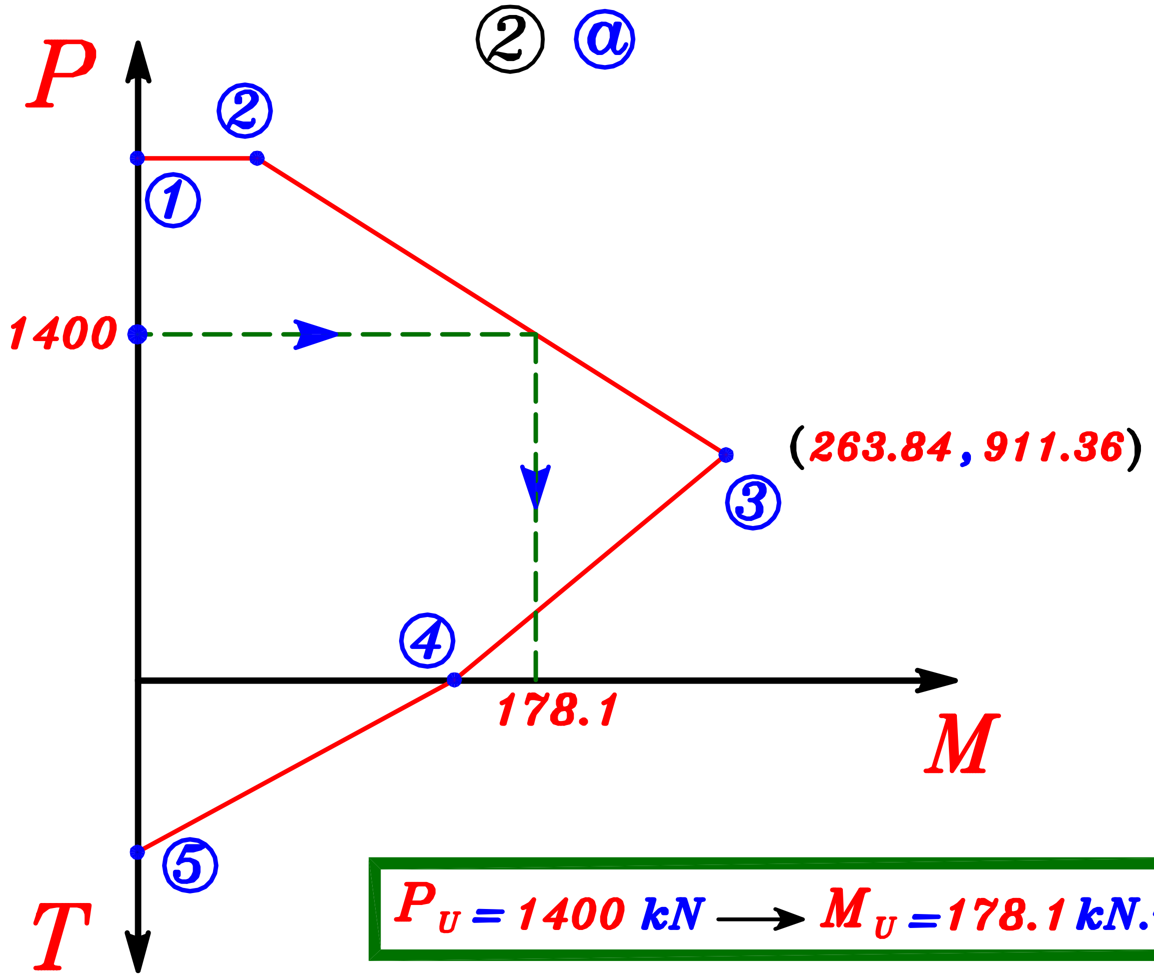
Point	$P_{U.L.}$ (kN)	$M_{U.L.}$ (kN.m)
①	2113.68	Zero
②	2113.68	52.84
③	911.36	263.84
④	Zero	139.82
⑤	− 669.1	Zero

① Interaction Diagram

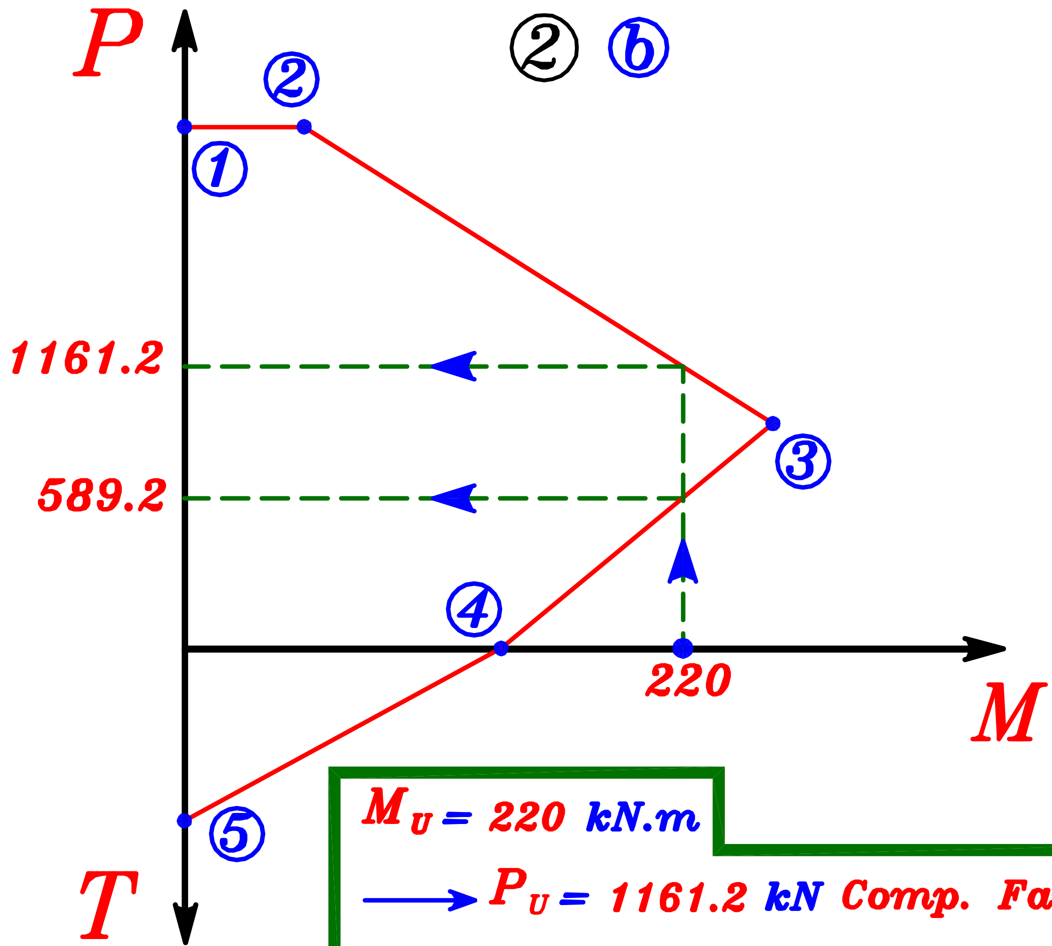


ملحوظه

مقياس الرسم الأفقى مختلف عن مقياس الرسم الرأسى



$P_U = 1400\ kN \longrightarrow M_U = 178.1\ kN.m$

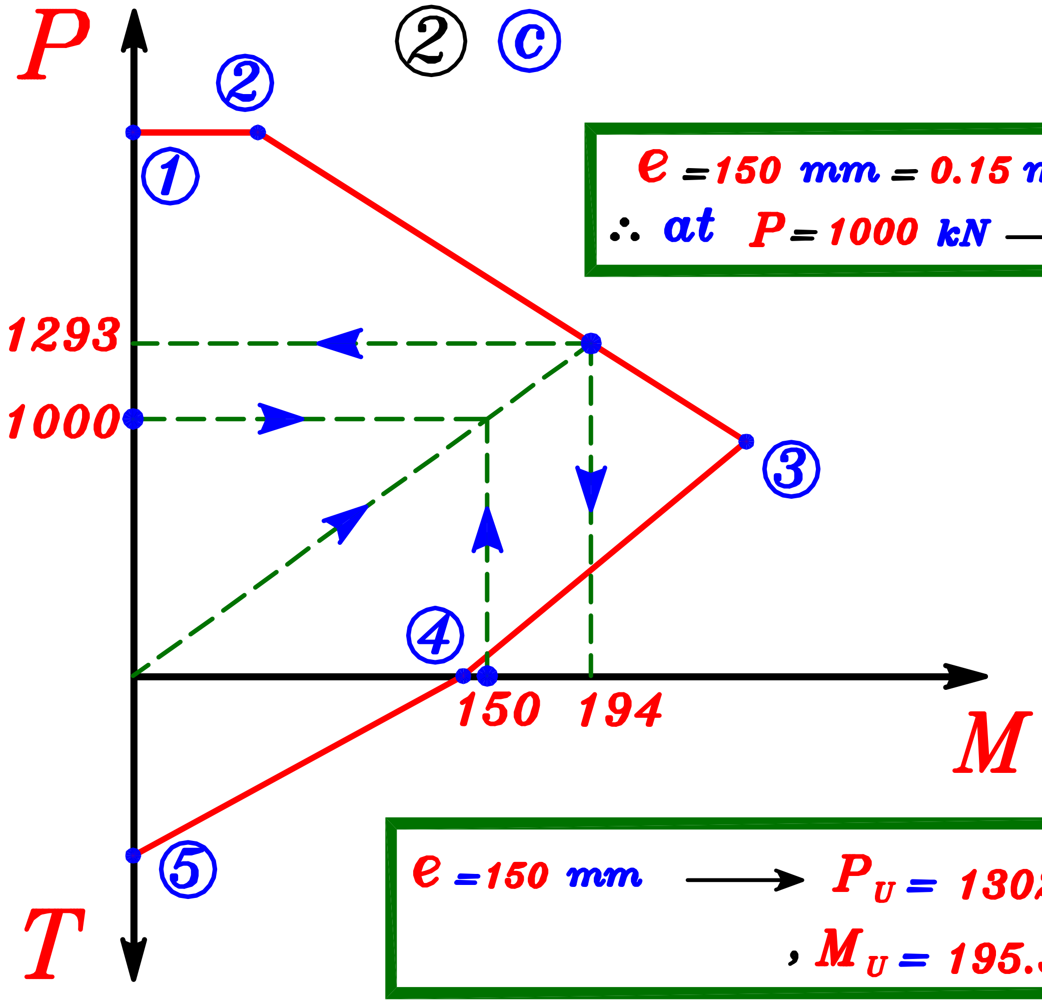


$M_U = 220\ kN.m$

$\longrightarrow P_U = 1161.2\ kN$ Comp. Failure

$\longrightarrow P_U = 589.2\ kN$ Ten. Failure

② ©



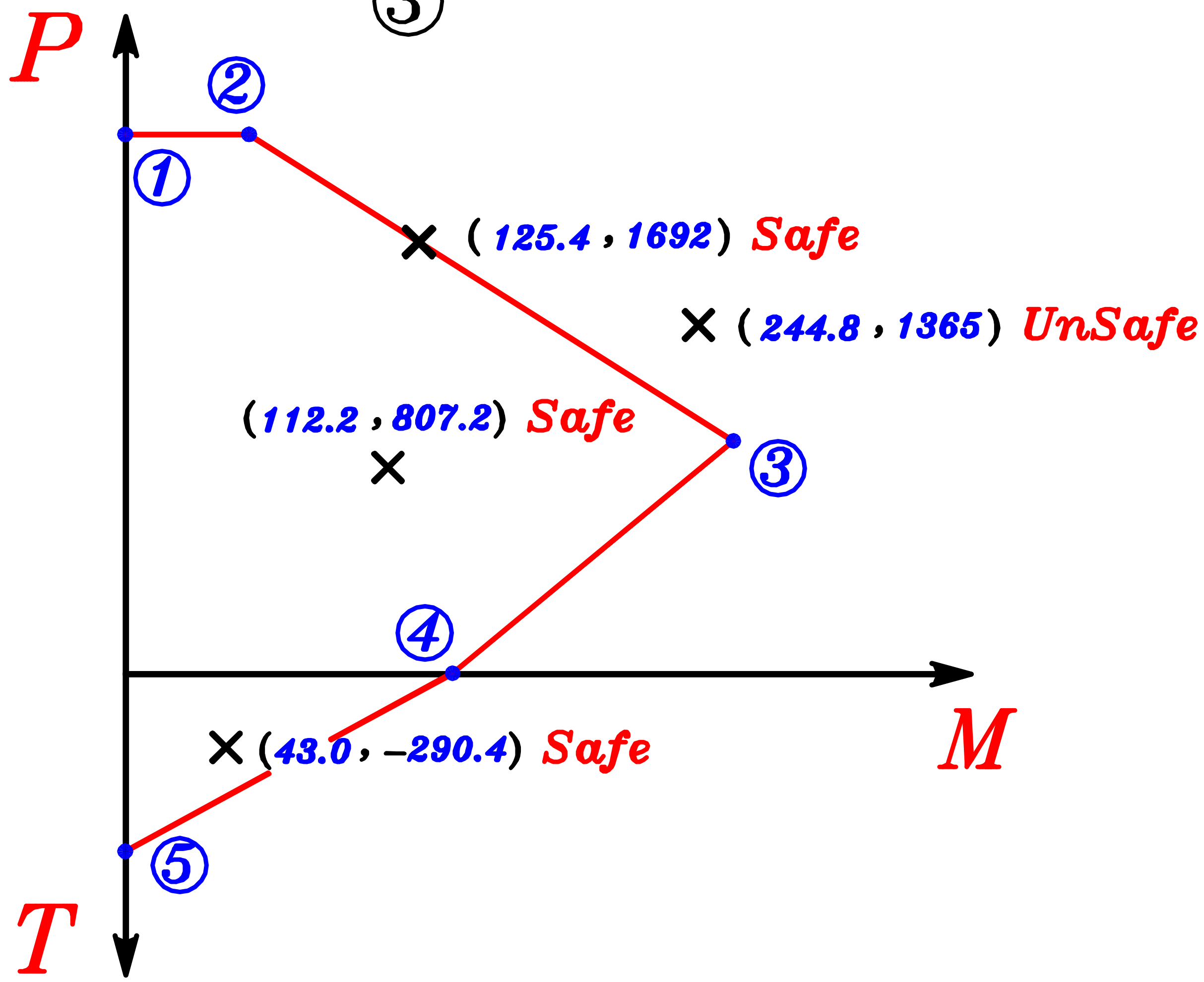
$e = 150\ mm = 0.15\ m$

$\therefore$ at $P = 1000\ kN \longrightarrow M = 150\ kN.m$

$e = 150\ mm \longrightarrow P_U = 1302\ kN$

$, M_U = 195.3\ kN.m$

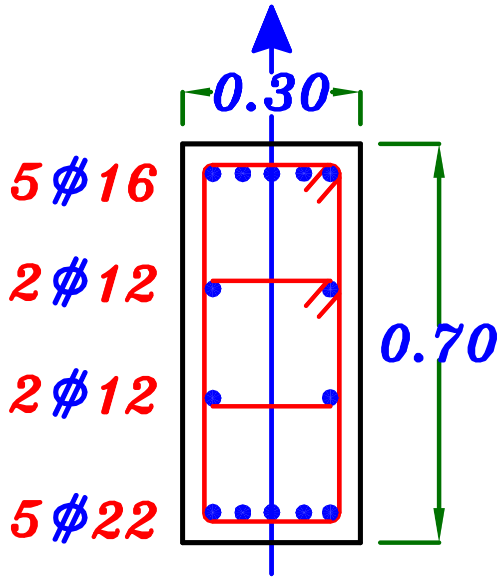
③



Example.

$F_{cu} = 30 \ \ N\backslash mm^2$

$F_y = 360 \ \ N\backslash mm^2$



Req.

Draw the Interaction Diagram

(Using Five points only)

Solution.

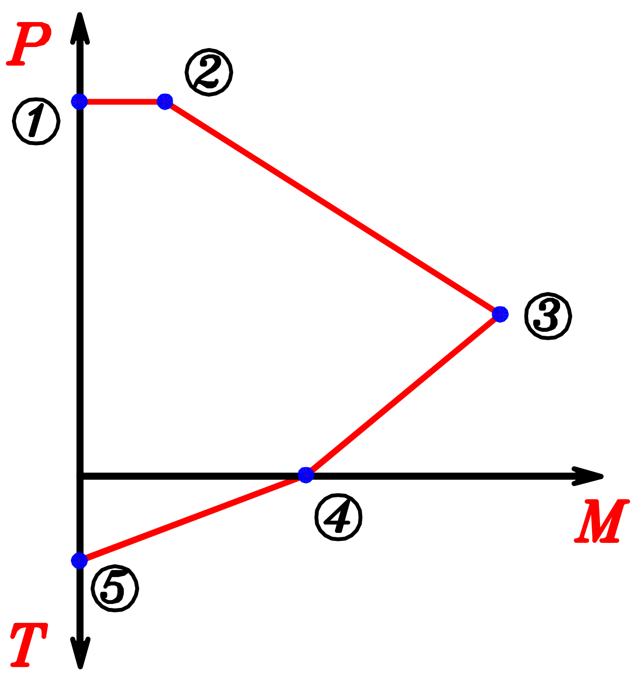
$A_s = 5\phi 22 = 1900 \ mm^2 \quad , \quad A_s' = 5\phi 16 = 1005 \ mm^2$

$side \ \ bars = 4\phi 12 = 452 \ mm^2$ مساحه الحديد الجانبى

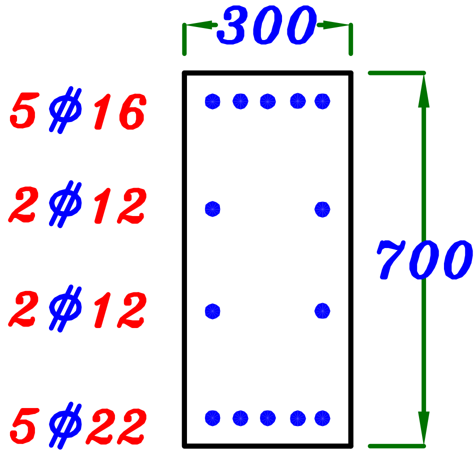
$A_{s_{total}} = 1900 + 1005 + 452 = 3357 \ mm^2$

للتسهيل لن نحسب plastic centroid و سوف نأخذالعزوم حول نقطه المنتصف .

$$\boxed{X = \frac{t}{2} = 350 \, mm}$$



Point 1 $M = Zero$



$A_c = 300 * 700 = 210000\ mm^2$

$A_s = (A_s + A_{s`} + side\ \ bars)$

$= 1900 + 1005 + 452 = 3357\ mm^2$

$P = P_{U.L.} = 0.35\ A_c\ F_{cu} + 0.67\ A_s\ F_y$

$= 0.35\ (210000)(30) + 0.67\ (3357)\ (360) = 3014708\ N$

$= 3014.7\ kN$

$$\boxed{P_{U.L.} = 3014.7\ kN\ \ ,\ \ M_{U.L.} = Zero}$$

Point 2 min. eccentricity $e_{min.} = 0.05\ t$

$P = P_{U.L.} = 0.35\ A_c\ F_{cu} + 0.67\ A_s\ F_y = 3014708\ N$

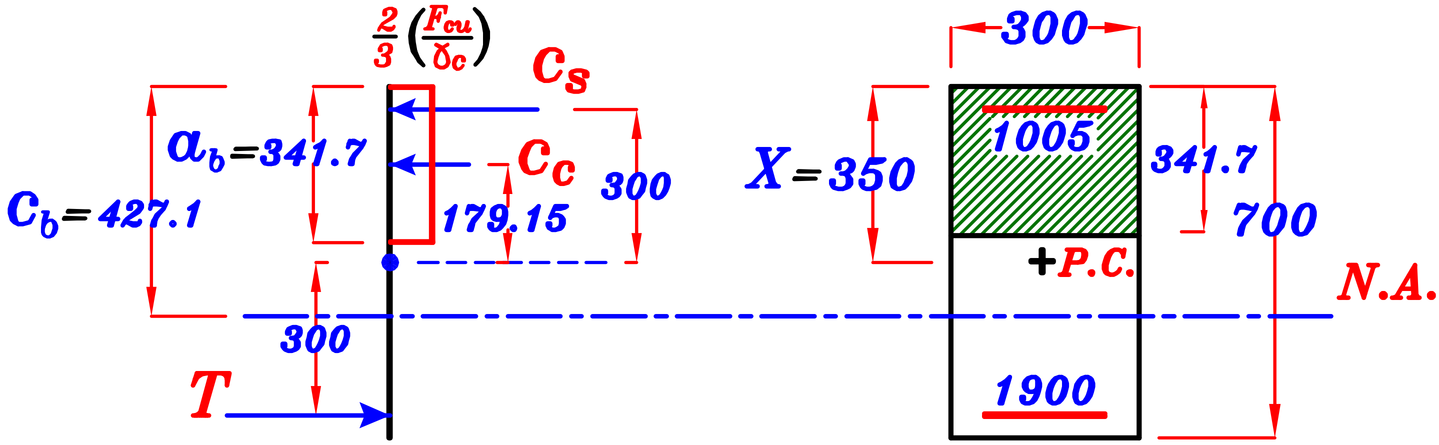
$= 3014.7\ kN$

$M = P_{U.L.} * e_{min.} = P_{U.L.} * 0.05\ t = 3014708 * 0.05(700)$

$= 105514780\ N.mm = 105.51\ kN.m$

$$\boxed{P_{U.L.} = 3014.7\ kN\ \ ,\ \ M_{U.L.} = 105.51\ kN.m}$$

Point ③ (Balanced Failure)



$$C_b = \left(\frac{600}{600 + (F_y \backslash \gamma_s)}\right) * d = \left(\frac{600}{600 + \left(\frac{360}{1.15}\right)}\right) * 650 = 427.14 \text{ mm}$$

Take $C = C_b = 427.14$ mm $\quad a_b = 0.8\, C_b = 341.7$ mm

$$\boxed{P = P_{U.L.} = C_c + C_s - T = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a_b b + A_s' \frac{F_y}{\gamma_s} - A_s \frac{F_y}{\gamma_s}}$$

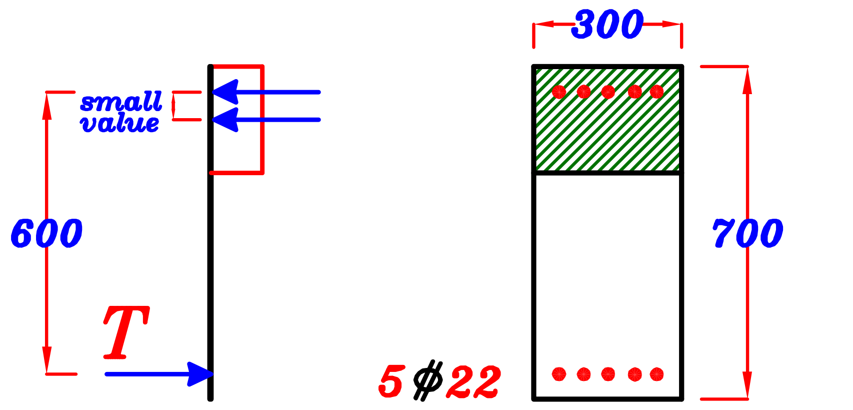
$$P = \frac{2}{3}\left(\frac{30}{1.5}\right)(341.7)(300) + 1005\left(\frac{360}{1.15}\right) - 1900\left(\frac{360}{1.15}\right) = 1086626 \text{ N}$$
$$= 1086.6 \text{ kN}$$

$$\boxed{M = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a_b b\left(X - \frac{a_b}{2}\right) + A_s' \frac{F_y}{\gamma_s}(X - d') + A_s \frac{F_y}{\gamma_s}(d - X)}$$

$$M = \frac{2}{3}\left(\frac{30}{1.5}\right)(341.7)(300)(179.15) + 1005\left(\frac{360}{1.15}\right)(300) + 1900\left(\frac{360}{1.15}\right)(300)$$
$$= 517679611 \text{ N.mm} = 517.67 \text{ kN.m}$$

$$\boxed{P_b = 1086.6 \text{ kN},\quad M_b = 517.67 \text{ kN.m}}$$

Point ④

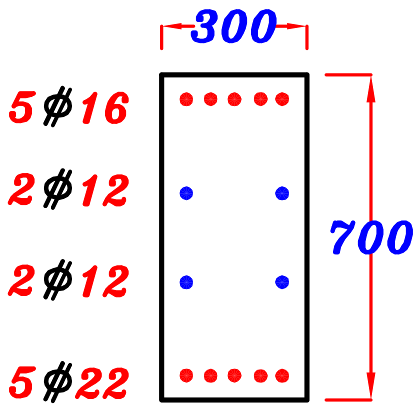


$$P = Zero$$

$$M = A_s \frac{F_y}{\gamma_s}(d - d') = 1900 * \frac{360}{1.15}(650 - 50) = 356869565 \ N.mm$$
$$= 356.87 \ kN.m$$

$$\boxed{P_{U.L.} = Zero \quad , \quad M_{U.L.} = 356.87 \ kN.m}$$

Point ⑤ $\quad M = Zero$

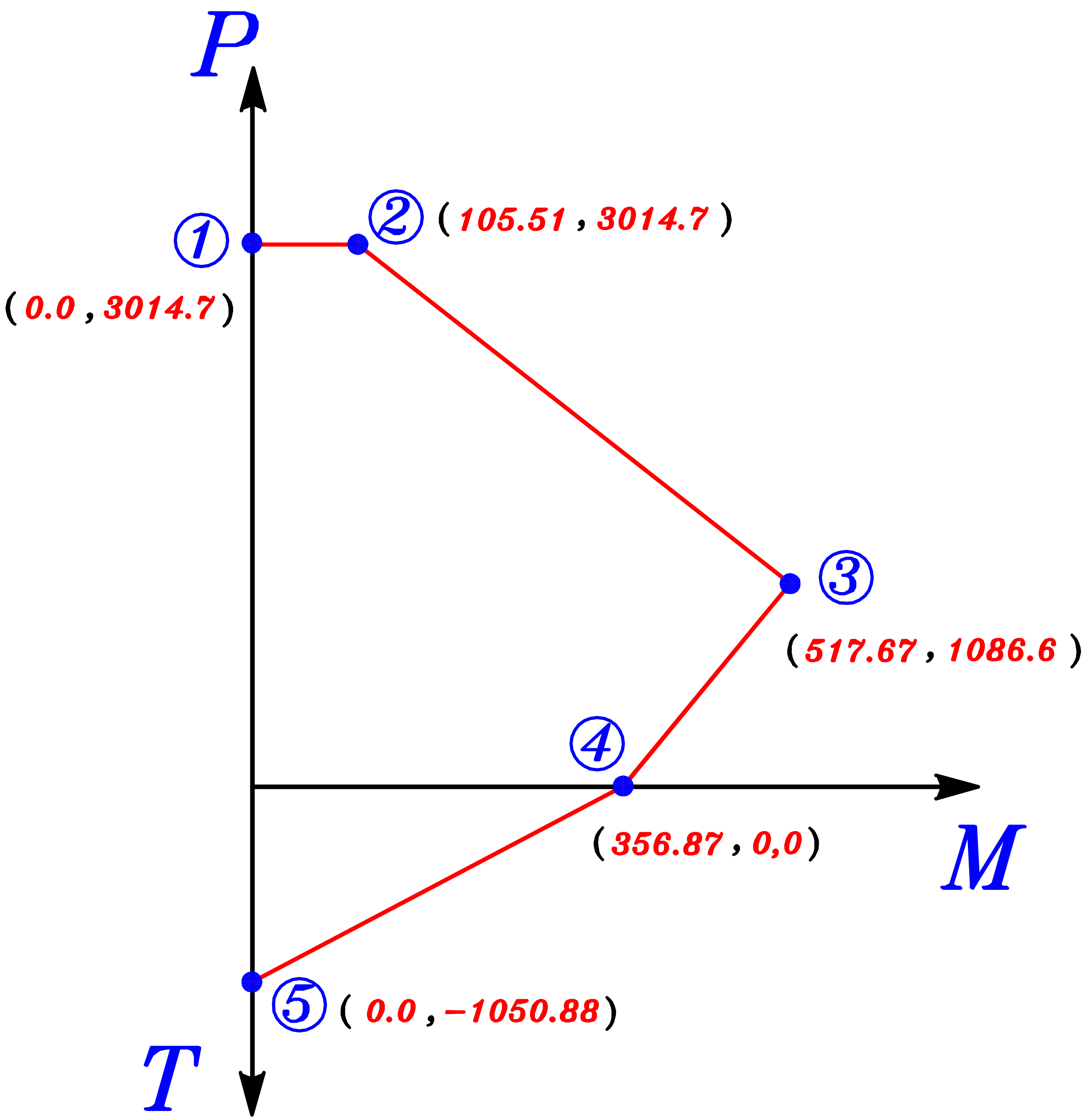


$$A_s = (A_s + A_s' + side \ \ bars)$$
$$= 1900 + 1005 + 452 = 3357 \ mm^2$$

$$T = A_s * \frac{F_y}{\gamma_s} = 3357 * \frac{360}{1.15} = 1050886.9 \ N = 1050.88 \ kN$$

$$\boxed{T_{U.L.} = 1050.88 \ kN \quad , \quad M_{U.L.} = Zero}$$

Point	$P_{U.L.}$ (kN)	$M_{U.L.}$ (kN.m)
①	3014.7	Zero
②	3014.7	105.51
③	1086.6	517.67
④	Zero	356.87
⑤	−1050.88	Zero



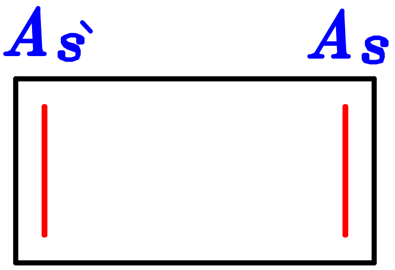
ملحوظه

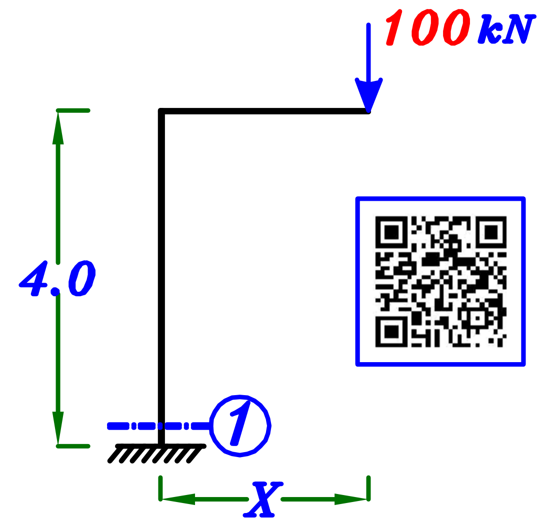
مقياس الرسم الأُفقى مختلف عن مقياس الرسم الرأسى

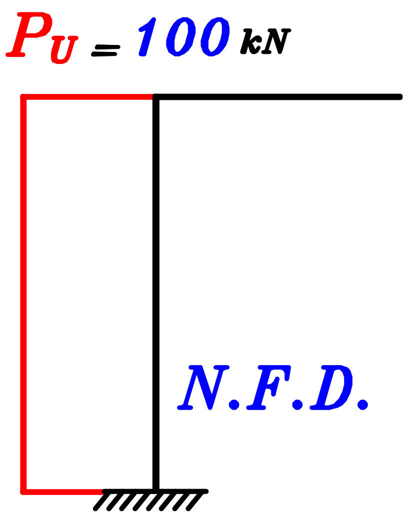
Example.

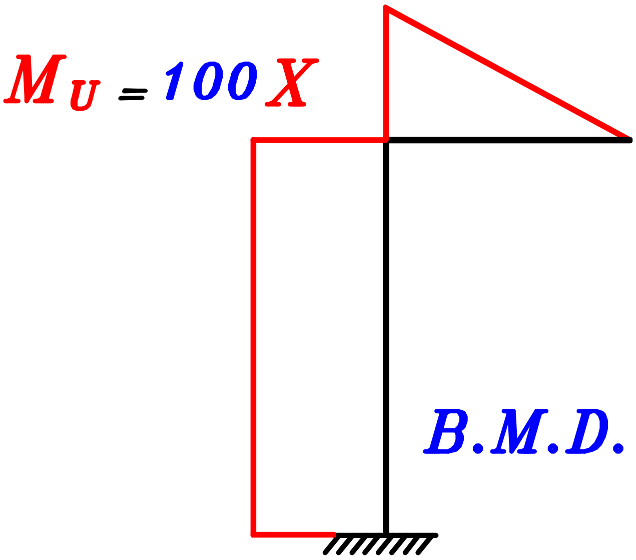
Calculate max. X

Sec. ①



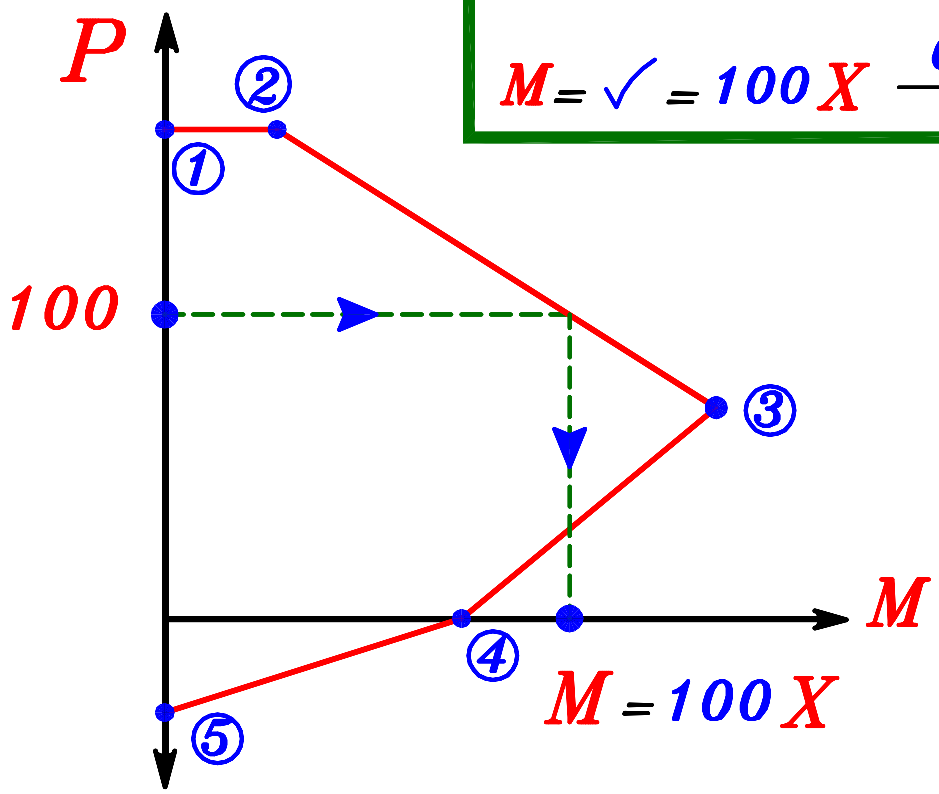






at $P = 100$ kN $\xrightarrow{\text{Get}}$ $M = \checkmark$

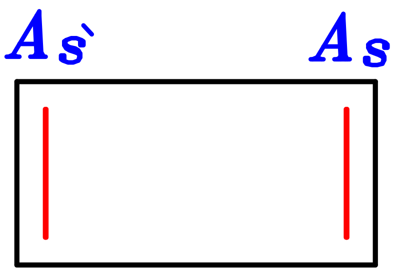
$M = \checkmark = 100\,X$ $\xrightarrow{\text{Get}}$ $X = \checkmark$

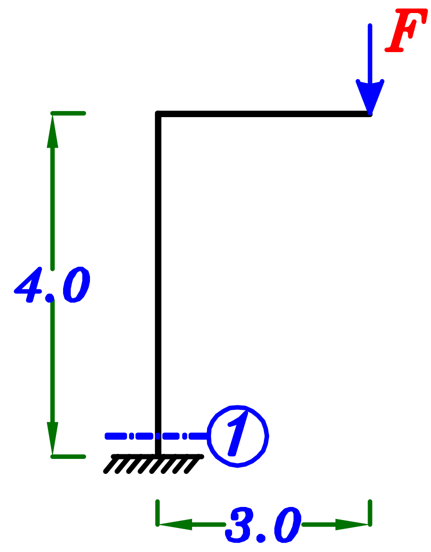


Example.

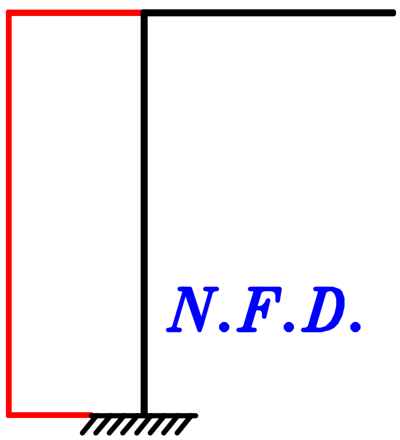
Calculate max. F

Sec. ①

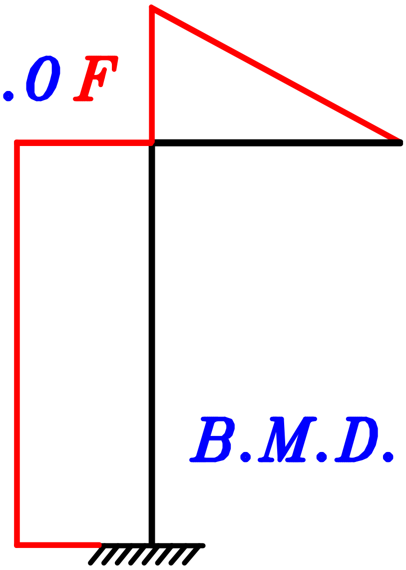




$P_U = F$



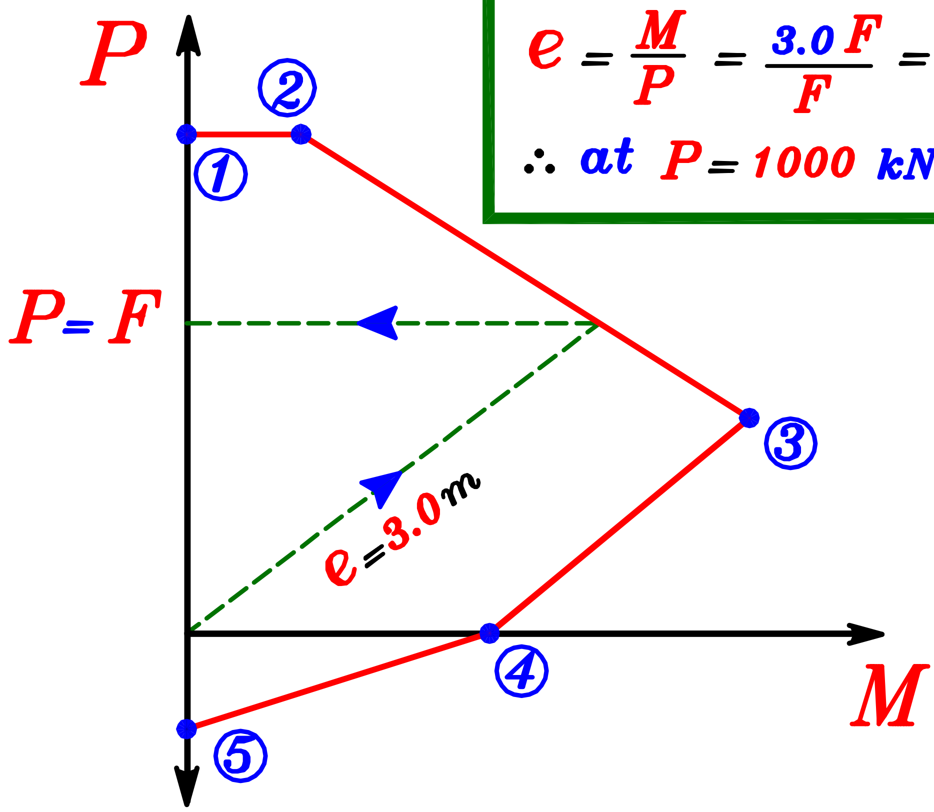
$M_U = 3.0\,F$



$P = F \quad , \quad M = 3.0\,F$

$$e = \frac{M}{P} = \frac{3.0\,F}{F} = 3.0\ m$$

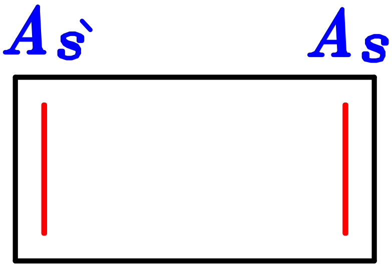
$\therefore$ at $P = 1000\ kN \longrightarrow M = 3000\ kN.m$

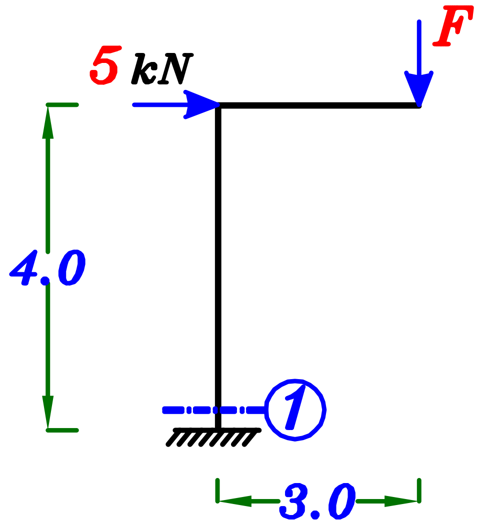


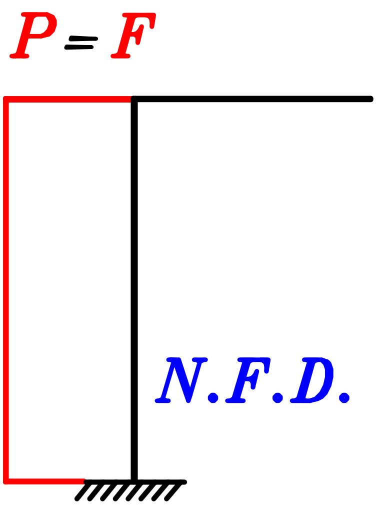
Example.

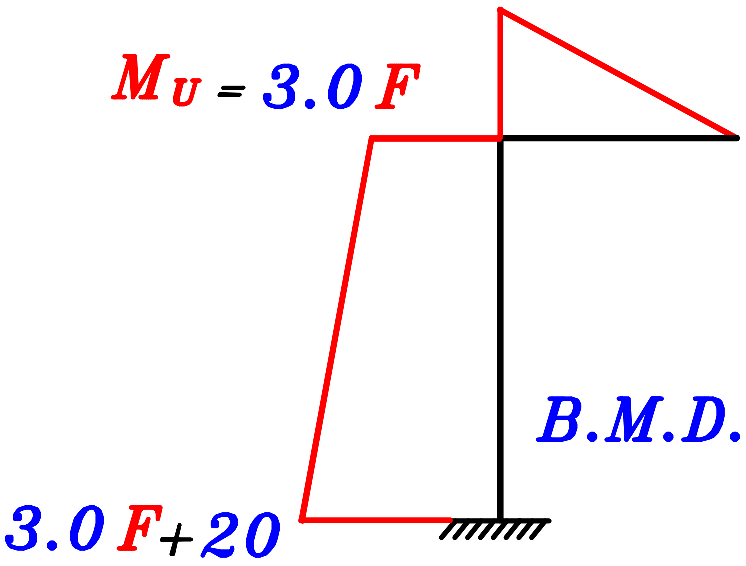
Calculate max. F

Sec. ①

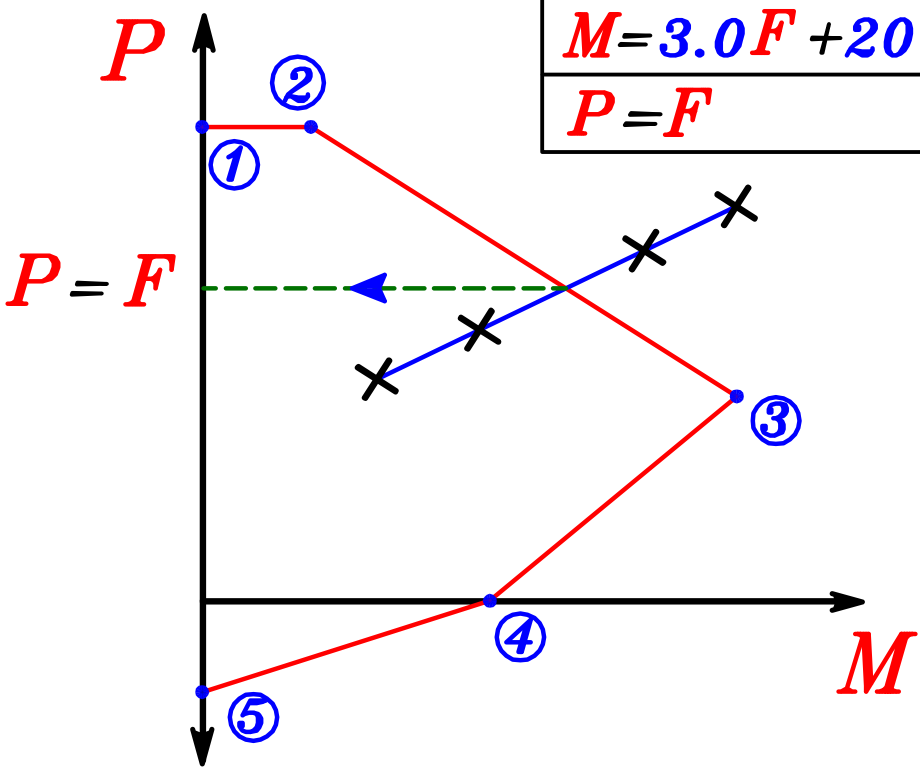






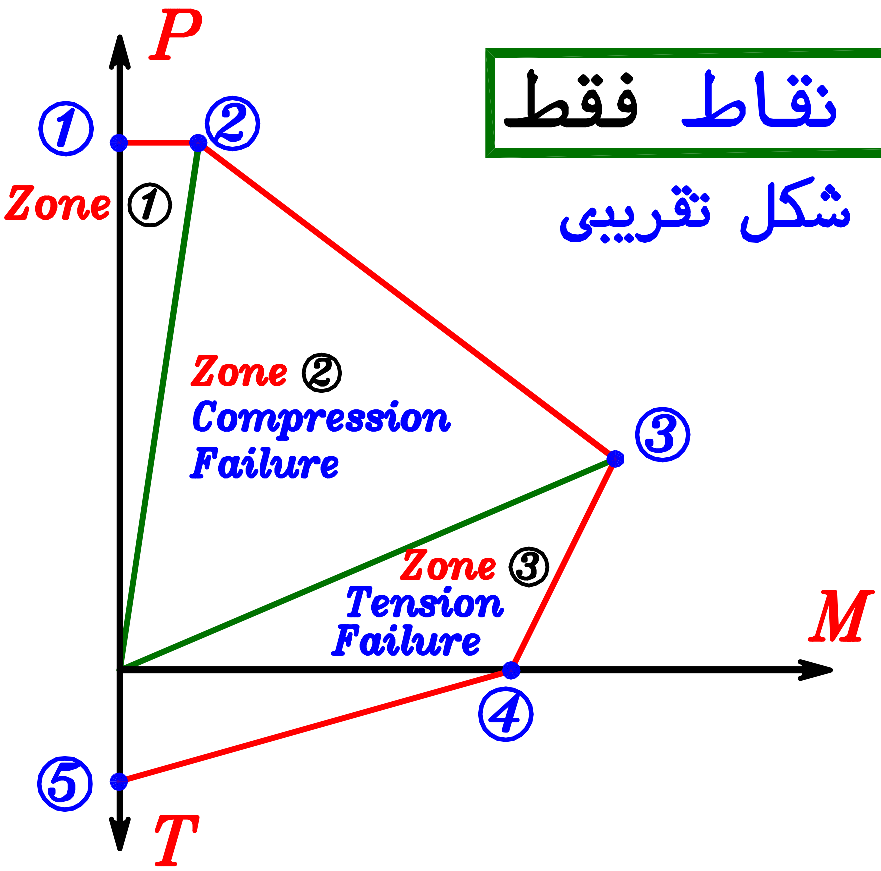


F	✓	✓	✓	
$M = 3.0F + 20$	✓	✓	✓	
$P = F$	✓	✓	✓	



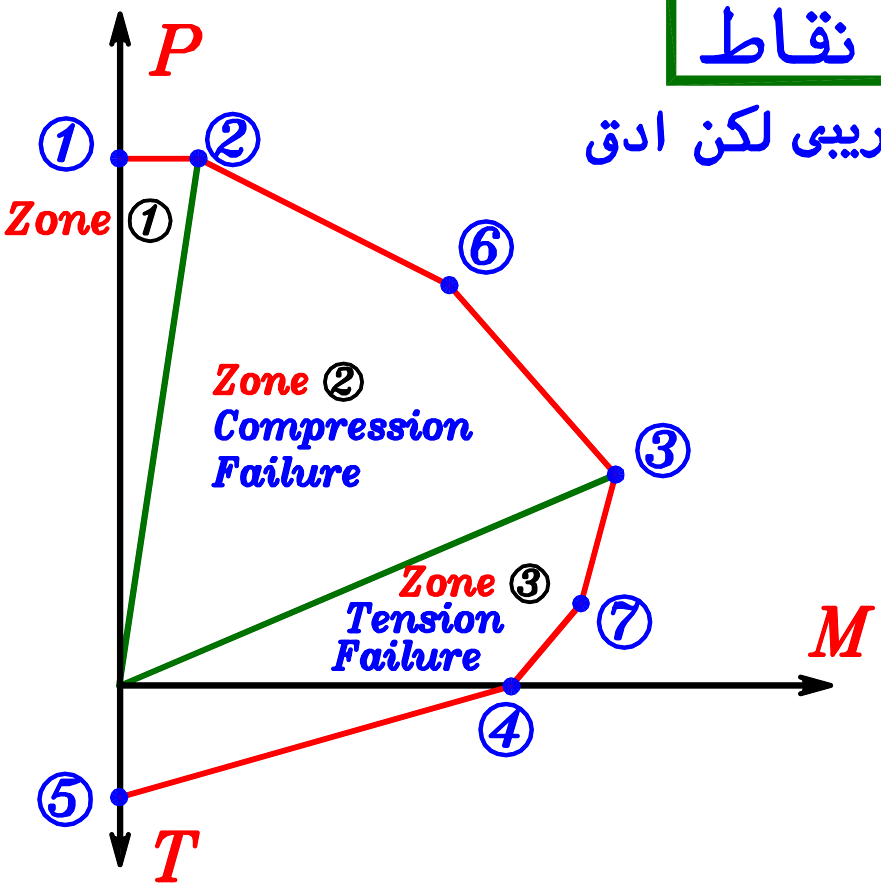
Drawing Exact Interaction Diagram.

By using Seven Points only.



٥ نقاط فقط Simplified

شكل تقريبى



٧ نقاط Exact

شكل تقريبى لكن ادق

To draw point ⑥ at Compression Failure Zone.

لان *point* ⑥ موجوده فى ال *Compression Failure Zone*

أى انه اذا حدث انهيار سيكون *Compression Failure*

$\therefore \; e < e_b \longrightarrow a > a_b$

$\therefore \; F_s` = \dfrac{F_y}{\gamma_s}$

$F_s \leqslant \dfrac{F_y}{\gamma_s}$

لانه *Compression Failure*

---

1 – Assume $e < e_b$

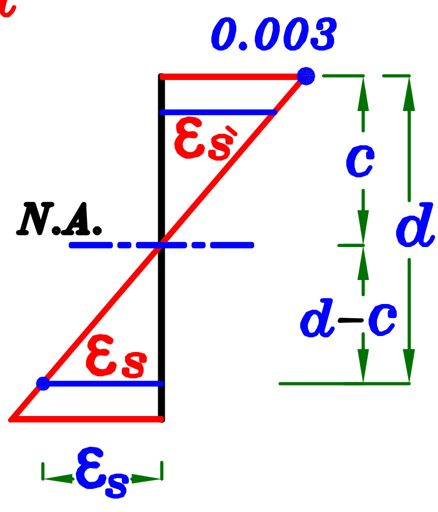
2 – Calculate

$$\boxed{\begin{aligned} \gamma_c &= 1.5\left[\left(\tfrac{7}{6}\right) - \tfrac{(e \backslash t)}{3}\right] \geqslant 1.5 \\ \gamma_s &= 1.15\left[\left(\tfrac{7}{6}\right) - \tfrac{(e \backslash t)}{3}\right] \geqslant 1.15 \end{aligned}}$$

3 – Choose $a > a_b = 0.8\left(\dfrac{600}{600 + (F_y \backslash \gamma_s)}\right) * d$

4 – Take $F_s` = \dfrac{F_y}{\gamma_s}$

5 – Calculate F_s



strain diagram

من شكل ال *strain diagram*

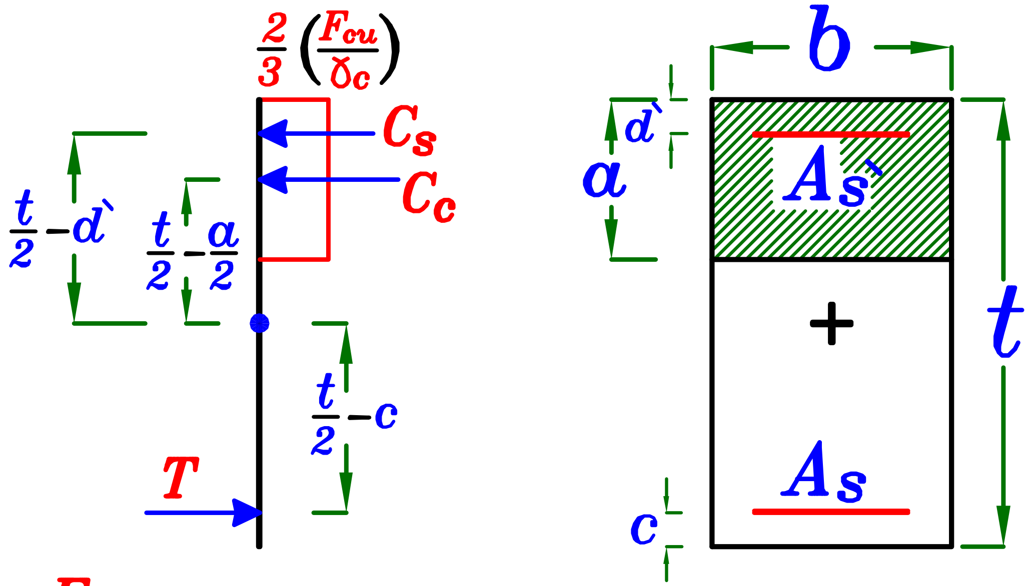
نحسب ε_s من تشابه المثلثات و نقارنها ε_y

$$\boxed{\varepsilon_y = \frac{F_y \backslash \gamma_s}{E_s} = \frac{F_y \backslash \gamma_s}{2 * 10^5}}$$

1– IF $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = \dfrac{F_y}{\gamma_s}$

2– IF $\varepsilon_s < \varepsilon_y \longrightarrow F_s < \dfrac{F_y}{\gamma_s} \longrightarrow F_s = \varepsilon_s * E_s$

$$\boxed{F_s = \varepsilon_s * 2 * 10^5}$$



$$C_c = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\,a\,b \quad , \quad C_s = A_s`\,\frac{F_y}{\gamma_s} \quad , \quad T = A_s\,F_s$$

$$\boxed{P = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\,a\,b + A_s`\,\frac{F_y}{\gamma_s} - A_s\,F_s}$$

$$\boxed{M = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\,a\,b\left(\frac{t}{2}-\frac{a}{2}\right) + A_s`\,\frac{F_y}{\gamma_s}\left(\frac{t}{2}-d`\right) + A_s\,F_s\left(\frac{t}{2}-c\right)}$$

تعتمد قيمه γ_c , γ_s على قيمه e

فنفرض قيمه لـ e و نحسب على اساسها قيمه كلا من γ_c , γ_s

ثم نحسب كلا من M, P ثم نعمل *check* على قيمه $e = \frac{M}{P}$ الفعليه .

و نقارنها بقيمه e المفروضه <u>فيكون كالاتى</u> :

١- اذا كانت قيمه e الفعليه قريبه من e المفروضه معناه ان قيمه M, P صحيحه و تكون احداثيات هذه النقطه صحيحه فى ال *Interaction Diagram* .

٢- اذا كانت قيمه e الفعليه بعيده عن e المفروضه معناه ان قيمه M, P غير صحيحه و يجب ان نفرض مره اخرى قيمه e و نحسب M, P مره اخرى ثم نعمل *check* مره اخرى وهكذا *trial & error* حتى تكون e الفعليه قريبه من e المفروضه .

فى الدراسه يفضل للتسهيل اعتبار $\gamma_c = 1.50 \; , \; \gamma_s = 1.15$

To draw point ⑦ at Tension Failure Zone.

لان point ⑦ موجوده فى ال Tension Failure Zone

أى انه اذا حدث انهيار سيكون Tension Failure

$\therefore\ e > e_b \longrightarrow a < a_b$

$\therefore\ F_s = \dfrac{F_y}{\gamma_s}$

$F_s` \leqslant \dfrac{F_y}{\gamma_s}$

لانه Tension Failure

1 - Assume $e > e_b$

2 - Calculate

$$\boxed{\begin{aligned}\gamma_c &= 1.5\left[\left(\tfrac{7}{6}\right) - \tfrac{(e\backslash t)}{3}\right] \geqslant 1.5\\ \gamma_s &= 1.15\left[\left(\tfrac{7}{6}\right) - \tfrac{(e\backslash t)}{3}\right] \geqslant 1.15\end{aligned}}$$

3 - Choose $a < a_b = 0.8\left(\dfrac{600}{600 + (F_y \backslash \gamma_s)}\right) * d$

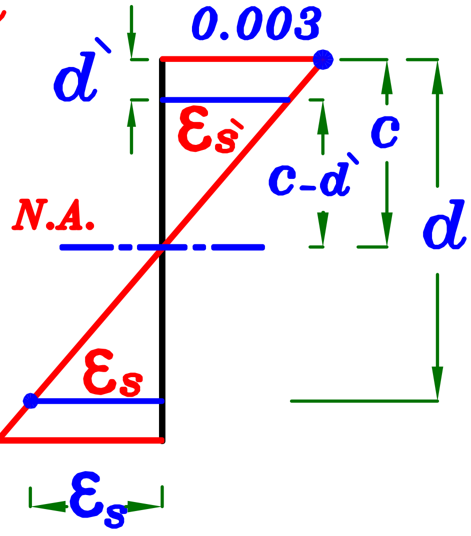
4 - Take $F_s = \dfrac{F_y}{\gamma_s}$

5 - Calculate $F_s`$

من شكل ال strain diagram

نحسب $\varepsilon_s`$ من تشابه المثلثات و نقارنها ε_y

$$\boxed{\varepsilon_y = \frac{F_y \backslash \gamma_s}{E_s} = \frac{F_y \backslash \gamma_s}{2 * 10^5}}$$

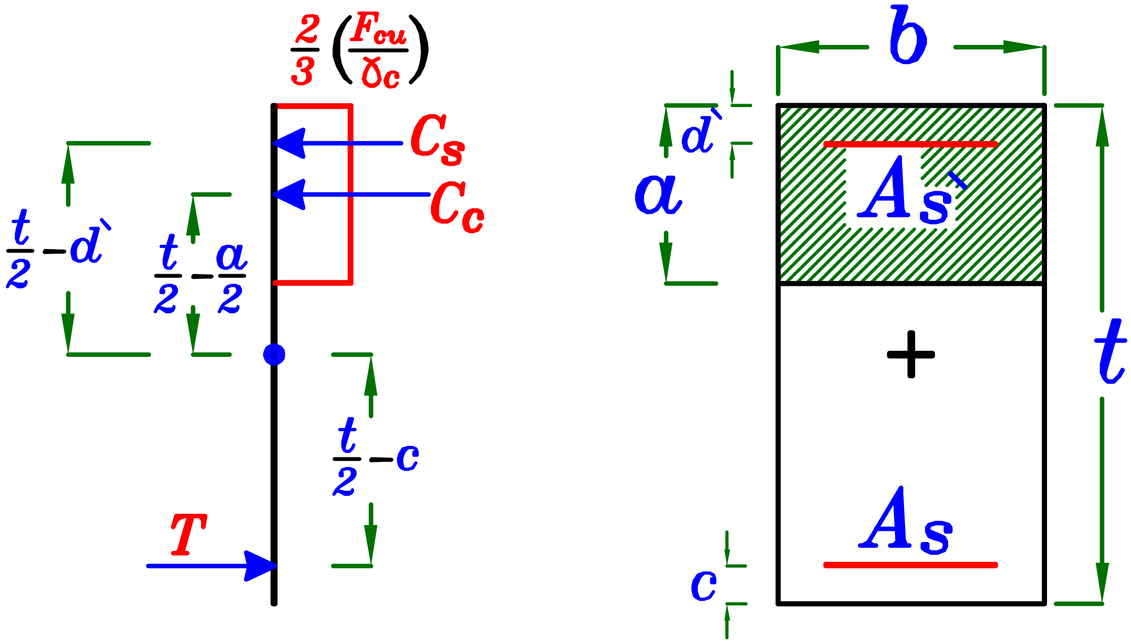


strain diagram

1- IF $\varepsilon_s` \geqslant \varepsilon_y \longrightarrow F_s` = \dfrac{F_y}{\gamma_s}$

2- IF $\varepsilon_s` < \varepsilon_y \longrightarrow F_s` < \dfrac{F_y}{\gamma_s} \longrightarrow F_s` = \varepsilon_s` * E_s$

$$\boxed{F_s` = \varepsilon_s` * 2 * 10^5}$$



$$C_c = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\,a\,b \quad , \quad C_s = A_{s`}\,F_{s`} \quad , \quad T = A_s\,\frac{F_y}{\gamma_s}$$

$$\boxed{P = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\,a\,b + A_{s`}\,F_{s`} - A_s\,\frac{F_y}{\gamma_s}}$$

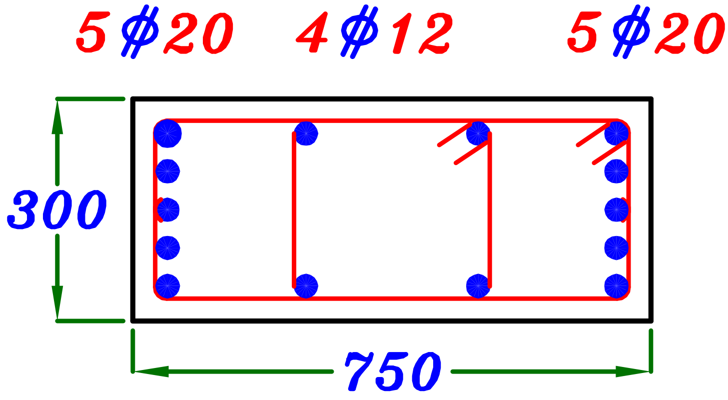
$$\boxed{M = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\,a\,b\left(\frac{t}{2}-\frac{a}{2}\right) + A_{s`}\,F_{s`}\left(\frac{t}{2}-d`\right) + A_s\,\frac{F_y}{\gamma_s}\left(\frac{t}{2}-c\right)}$$

في الدراسه يفضل للتسهيل اعتبار $\gamma_c = 1.50$, $\gamma_s = 1.15$

Example.

$F_{cu} = 30\ N\backslash mm^2$

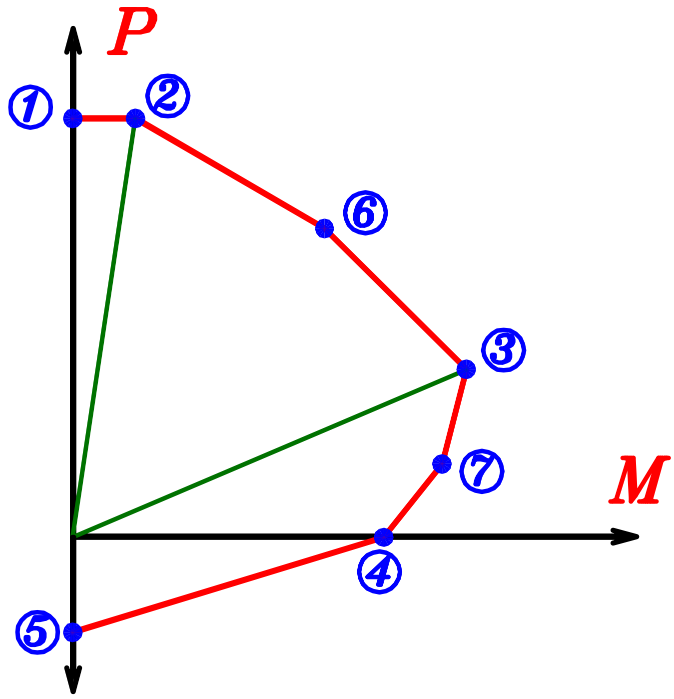
$F_y = 360\ N\backslash mm^2$



Req.

Draw the exact interaction diagram using 7 points.

Solution.

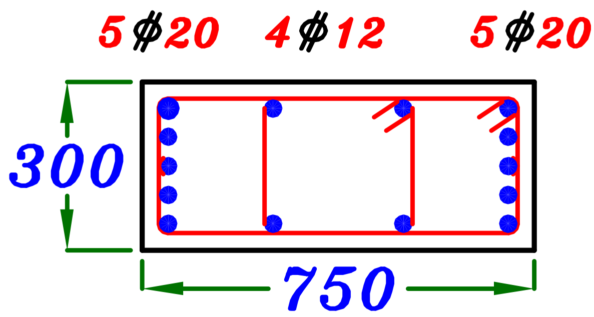


$A_c = 300 * 750 = 225000\ mm^2$

$A_s = A_s` = 5\phi20 = 1570\ mm^2$

$A_{s\,total} = 5\phi20 + 5\phi20 + 4\phi12$

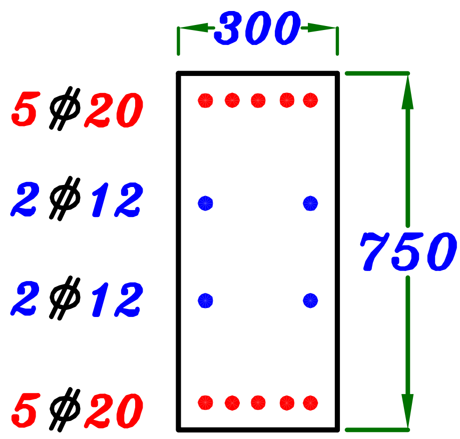
$= 3592\ mm^2$



$$C_b = \left(\frac{600}{600 + (F_y \backslash \gamma_s)}\right) * d = \left(\frac{600}{600 + \left(\frac{360}{1.15}\right)}\right) * 700 = 460\ mm$$

$$a_b = 0.8\ C_b = 0.8 * 460 = 368\ mm$$

Point ① $M = Zero$



$$A_c = 300 * 750 = 225000 \ mm^2$$

$$As_{total} = 5\phi20 + 5\phi20 + 4\phi12 = 3592 \ mm^2$$

$$P = P_{U.L.} = 0.35\, A_c\, F_{cu} + 0.67\, A_s\, F_y$$

$$P = 0.35\ (225000)(30) + 0.67\ (3592)\ (360) = 3228890 \ N$$
$$= 3228.9 \ kN$$

$$\boxed{P_{U.L.} = 3228.9 \ kN \quad , \quad M_{U.L.} = Zero}$$

Point ② min. eccentricity $e_{min.} = 0.05\ t$

$$P = P_{U.L.} = 0.35\, A_c\, F_{cu} + 0.67\, A_s\, F_y$$

$$= 0.35\ (225000)(30) + 0.67\ (3592)\ (360) = 3228890 \ N$$
$$= 3228.9 \ kN$$

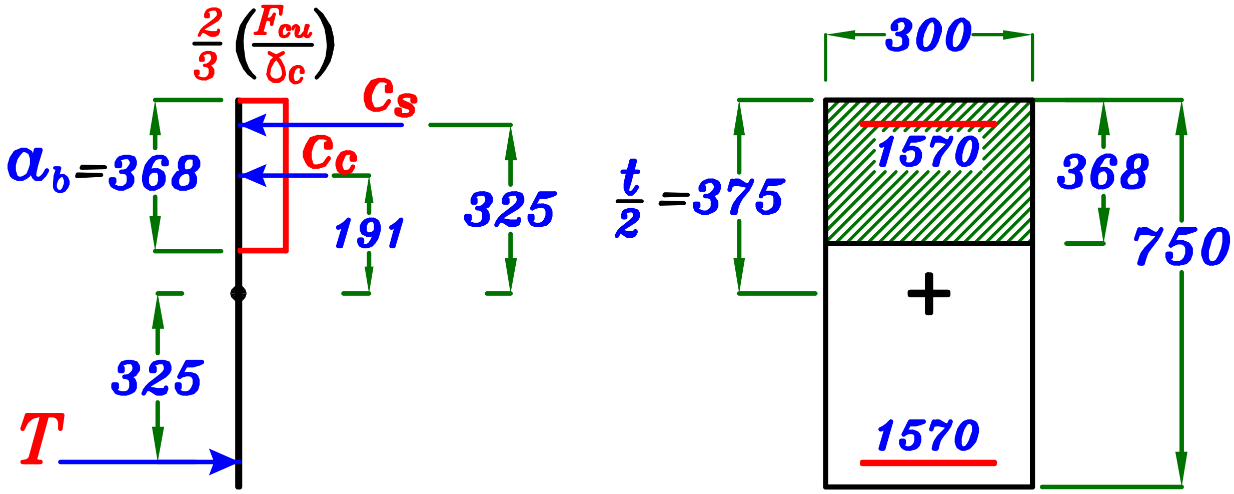
$$M = P_{U.L.} * e_{min.} = P_{U.L.} * 0.05\ t = 3228890 * 0.05(750)$$
$$= 121083375 \ N.mm = 121.1 \ kN.m$$

$$\boxed{P_{U.L.} = 3228.9 \ kN \quad , \quad M_{U.L.} = 121.1 \ kN.m}$$

Point ③ (Balanced Failure)

$$C_b = \left(\frac{600}{600 + (F_y \backslash \gamma_s)}\right) * d = \left(\frac{600}{600 + (\frac{360}{1.15})}\right) * 700 = 460\ mm$$

$$a_b = 0.8\ C_b = 0.8 * 492.8 = 368\ mm$$



$$\boxed{P_b = C_c + C_s - T = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a_b b + A_{s'}\frac{F_y}{\gamma_s} - A_s\frac{F_y}{\gamma_s}}$$

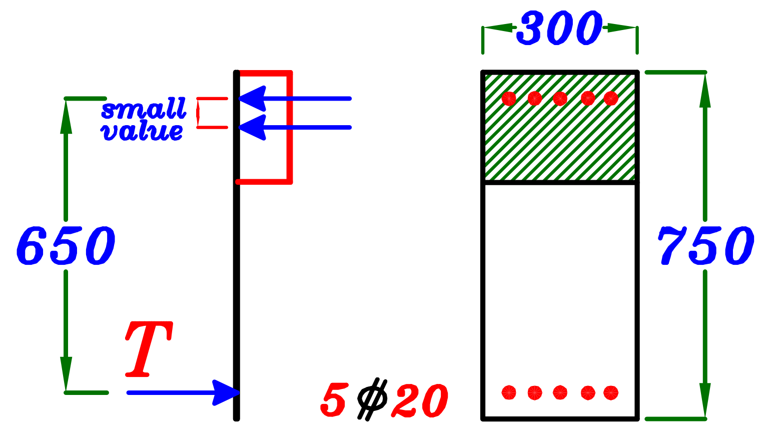
$$P_b = \frac{2}{3}\left(\frac{30}{1.5}\right)(368)(300) + 1570\left(\frac{360}{1.15}\right) - 1570\left(\frac{360}{1.15}\right) = 1472000\ N$$
$$= 1472\ kN$$

$$\boxed{M_b = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a_b b \left(\frac{t}{2} - \frac{a_b}{2}\right) + A_{s'}\frac{F_y}{\gamma_s}\left(\frac{t}{2} - d'\right) + A_s\frac{F_y}{\gamma_s}\left(\frac{t}{2} - c\right)}$$

$$M_b = \frac{2}{3}\left(\frac{30}{1.5}\right)(368)(300)(191) + 1570\left(\frac{360}{1.15}\right)(325) + 1570\left(\frac{360}{1.15}\right)(325)$$
$$= 600612869\ N.mm = 600.6\ kN.m$$

$$\boxed{P_b = 1472\ kN\ ,\quad M_b = 600.6\ kN.m}$$

Point ④



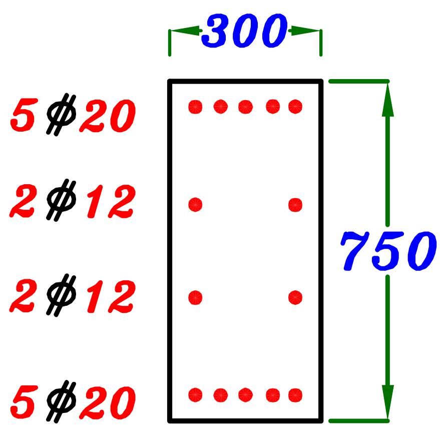
$A_s = 5\phi 20 = 1570 \ mm^2$

$P = Zero$

$M = A_s \frac{F_y}{\gamma_s}(d - d`) = 1570 * \frac{360}{1.15}(700 - 50) = 319460869 \ N.mm$
$= 319.46 \ kN.m$

$$\boxed{P_{U.L.} = Zero \quad , \quad M_{U.L.} = 319.46 \ kN.m}$$

Point ⑤ $\quad M = Zero$



$As_{total} = 5\phi 20 + 5\phi 20 + 4\phi 12$

$= 3592 \ mm^2$

$T = A_s * \frac{F_y}{\gamma_s} = 3592 * \frac{360}{1.15} = 1124452.1 \ N = 1124.4 \ kN$

$$\boxed{T_{U.L.} = 1124.4 \ kN \quad , \quad M_{U.L.} = Zero}$$

Point ⑥ (Compression Failure zone)

1 - assume $\gamma_c = 1.50$, $\gamma_s = 1.15$

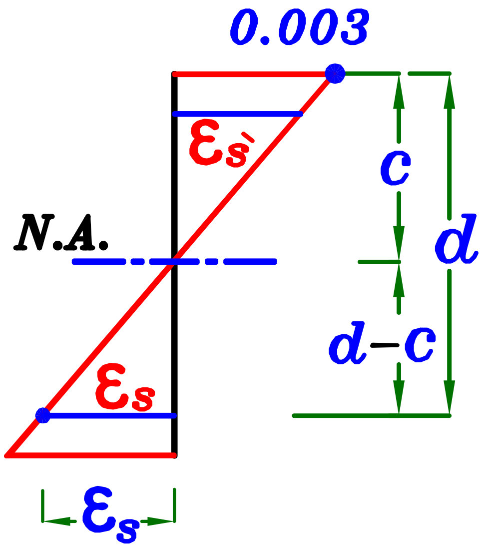
2 - Take $F_s' = \dfrac{F_y}{\gamma_s}$

3 - Choose $a > a_b$

4 - Calculate F_s

من شكل ال strain diagram

نحسب ε_s من تشابه المثلثات و نقارنها ε_y



strain diagram

$$\boxed{\varepsilon_y = \frac{F_y \backslash \gamma_s}{E_s} = \frac{F_y \backslash \gamma_s}{2 * 10^5}}$$

1- IF $\varepsilon_s \geq \varepsilon_y \longrightarrow F_s = \dfrac{F_y}{\gamma_s}$

2- IF $\varepsilon_s < \varepsilon_y \longrightarrow F_s < \dfrac{F_y}{\gamma_s} \longrightarrow F_s = \varepsilon_s * E_s$

$$\boxed{F_s = \varepsilon_s * 2 * 10^5}$$

1 - assume $\gamma_c = 1.50$, $\gamma_s = 1.15$

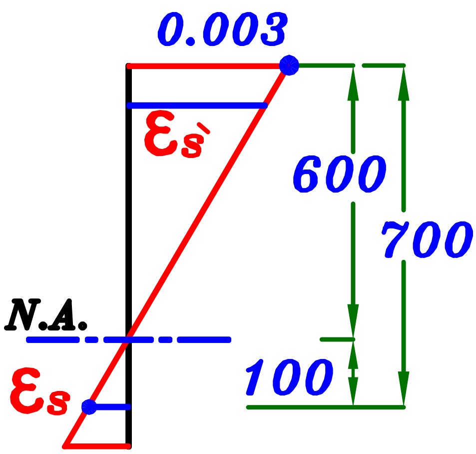
2 - Take $F_s' = \dfrac{F_y}{\gamma_s}$

3 - Choose $a > a_b$ Choose $\boxed{a = 480\ mm}$

$$\boxed{C = 1.25\,a = 600\ mm}$$

4 - Calculate F_s

$$\frac{0.003}{600} = \frac{\varepsilon_s}{100} \longrightarrow \boxed{\varepsilon_s = 0.0005}$$



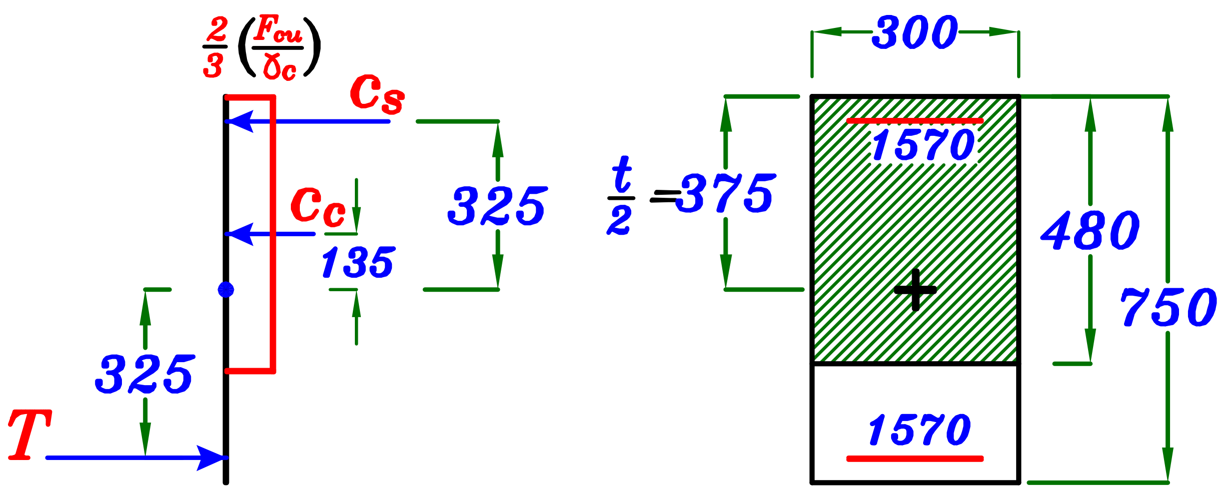
stain diagram

$$\varepsilon_y = \frac{F_y \backslash \gamma_s}{E_s} = \frac{360 \backslash 1.15}{2 * 10^5}$$

$$\boxed{\varepsilon_y = 0.00156}$$

$\therefore\ \varepsilon_s < \varepsilon_y$

$\therefore\ F_s = \varepsilon_s * 2 * 10^5 = 0.0005 * 2 * 10^5 = 100\ N/mm^2$



$$\boxed{P = C_c + C_s - T = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b + A_{s`}\frac{F_y}{\gamma_s} - A_s F_s}$$

$$P = \frac{2}{3}\left(\frac{30}{1.5}\right)(480)(300) + 1570\left(\frac{360}{1.15}\right) - 1570\,(100) = 2254478\ N$$

$$= 2254.4\ kN$$

$$\boxed{M = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b\left(\frac{t}{2} - \frac{a}{2}\right) + A_{s`}\frac{F_y}{\gamma_s}\left(\frac{t}{2} - d`\right) + A_s F_s\left(\frac{t}{2} - c\right)}$$

$$M = \frac{2}{3}\left(\frac{30}{1.5}\right)(480)(300)(135) + 1570\left(\frac{360}{1.15}\right)(325) + 1570\,(100)(325)$$

$$= 469955434\ N.mm$$

$$= 469.9\ kN.m$$

$$\boxed{P_{U.L.} = 2254.4\ kN \quad , \quad M_{U.L.} = 469.9\ kN.m}$$

Point 7 (Tension Failure zone)

1 – Take $\gamma_c = 1.50$, $\gamma_s = 1.15$

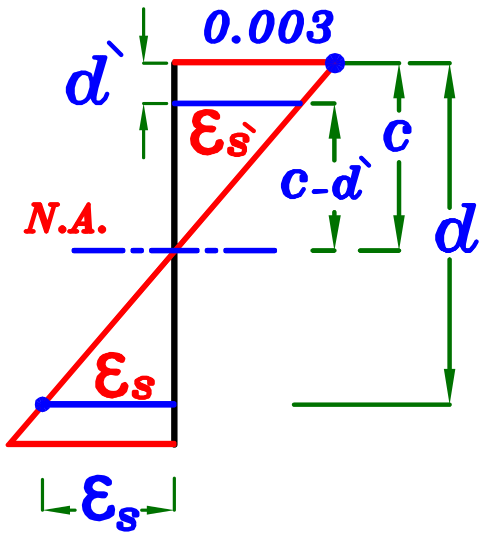
2 – Take $F_s = \dfrac{F_y}{\gamma_s}$

3 – Choose $a < a_b$

4 – Calculate $F_s`$

من شكل ال strain diagram

نحسب $\varepsilon_s`$ من تشابه المثلثات و نقارنها ε_y



stain diagram

$$\boxed{\varepsilon_y = \frac{F_y \backslash \gamma_s}{E_s} = \frac{F_y \backslash \gamma_s}{2 * 10^5}}$$

1– IF $\varepsilon_s` \geq \varepsilon_y \longrightarrow F_s` = \dfrac{F_y}{\gamma_s}$

2– IF $\varepsilon_s` < \varepsilon_y \longrightarrow F_s` < \dfrac{F_y}{\gamma_s} \longrightarrow F_s` = \varepsilon_s` * E_s$

$$\boxed{F_s` = \varepsilon_s` * 2 * 10^5}$$

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1 – Take $\gamma_c = 1.50$, $\gamma_s = 1.15$

2 – Take $F_s = \dfrac{F_y}{\gamma_s}$

3 – Choose $a < a_b$ Choose $\boxed{a = 240\ mm}$

$$\boxed{C = 1.25\, a = 300\ mm}$$

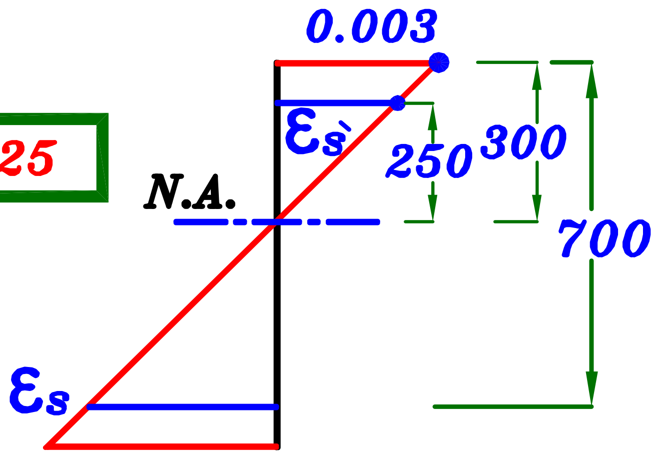
4 - Calculate $F_{s`}$

$$\frac{0.003}{300} = \frac{\varepsilon_{s`}}{250} \longrightarrow \boxed{\varepsilon_{s`} = 0.0025}$$

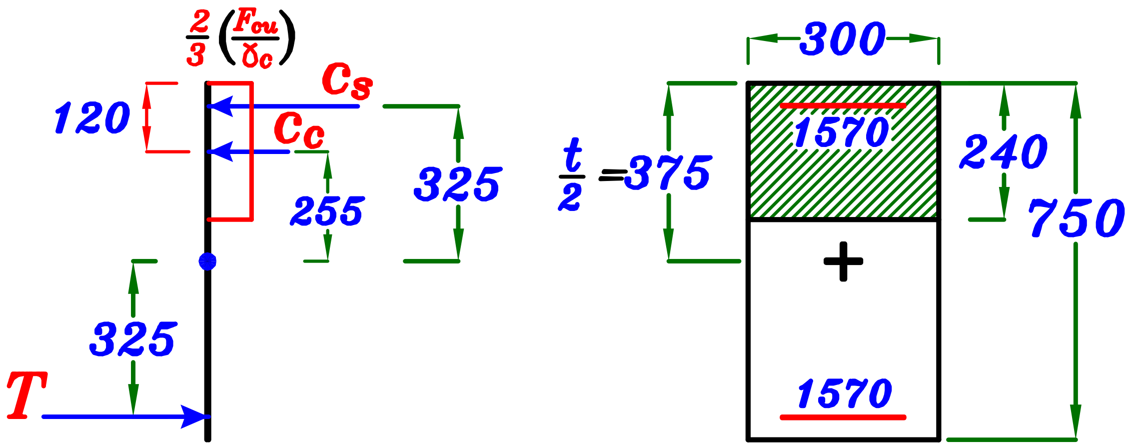
$$\varepsilon_y = \frac{F_y \backslash \gamma_s}{E_s} = \frac{360 \backslash 1.15}{2 * 10^5}$$

$$\boxed{\varepsilon_y = 0.00156}$$

$$\therefore \ \varepsilon_{s`} > \varepsilon_y \longrightarrow F_{s`} = \frac{F_y}{\gamma_s}$$



stain diagram



$$\boxed{P = C_c + C_s - T = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a \ b + A_{s`} F_{s`} - A_s \frac{F_y}{\gamma_s}}$$

$$P = \frac{2}{3}\left(\frac{30}{1.5}\right)(240)(300) + 1570\left(\frac{360}{1.15}\right) - 1570\left(\frac{360}{1.15}\right) = 960000 \ N$$

$$= 960.0 \ kN$$

$$\boxed{M = \frac{2}{3}\frac{F_{cu}}{\gamma_c} a \ b \left(\frac{t}{2} - \frac{a}{2}\right) + A_{s`} F_{s`} \left(\frac{t}{2} - d`\right) + A_s \frac{F_y}{\gamma_s}\left(\frac{t}{2} - c\right)}$$

$$M = \frac{2}{3}\left(\frac{30}{1.5}\right)(240)(300)(255) + 1570\left(\frac{360}{1.15}\right)(325) + 1570\left(\frac{360}{1.15}\right)(325)$$

$$= 564260869 \ N.mm = 564.2 \ kN.m$$

$$\boxed{P_{U.L.} = 960.0 \ kN \quad , \quad M_{U.L.} = 564.2 \ kN.m}$$

Point	P	M
1	3228.9	Zero
2	3228.9	121.1
3	1472	600.6
4	Zero	319.46
5	− 1124.4	Zero
6	2254.4	469.9
7	960.0	564.2

ملحوظه مقياس الرسم الأفقى مختلف عن مقياس الرسم الرأسى